

MODEL REFERENCE ADAPTIVE CONTROL OF
A NUCLEAR ROCKET ENGINE

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A DISSERTATION PRESENTED TO THE GRADUATE COUNCIL OF
THE UNIVERSITY OF FLORIDA
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE
DEGREE OF DOCTOR OF PHILOSOPHY

UNIVERSITY OF FLORIDA

April, 1966



DEDICATION

To my wife, Mildred,
my son, David,
my daughter, Jacqueline,
and my mother,
for their unfailing
understanding and encouragement

ACKNOWLEDGMENTS

The author wishes to express his gratitude to the co-chairmen of his supervisory committee, Dr. Robert E. Uhrig and Dr. Andrew P. Sage, who suggested this work and offered continued help and encouragement throughout its development. The demonstrated concern and interest of the remaining members of the committee, Professor Glen J. Schoessow, Dr. William H. Ellis, Dr. Walter Mauderli, and Dr. Robert G. Blake, had a most beneficial effect upon the work's progress and is sincerely appreciated.

Special thanks go to Mr. Kenneth Fawcett for his inspired maintenance of the analog computing equipment, to Mr. Reese Waters for his help with the MIMIC code, and to Mr. Lawrence Fitzgerald for his assistance with some of the digital computation. Discussions with my good friend Mr. Jagdish Saluja provided some of the most rewarding moments of the investigation.

It is a pleasure to extend thanks to Mrs. Wilma Smith for her capable assistance in typing the manuscript and to Mr. Richard Schoessow for his execution

of the drawings. Finally, the author expresses his appreciation to the United States Air Force for providing this opportunity for graduate study at the University of Florida.

TABLE OF CONTENTS

	Page
DEDICATION	ii
ACKNOWLEDGMENTS	iii
LIST OF TABLES	vii
LIST OF FIGURES	viii
LIST OF SYMBOLS	xi
ABSTRACT	xiii
CHAPTER	
I. INTRODUCTION	1
II. THE NUCLEAR ROCKET ENGINE	10
III. ADAPTIVE CONTROL FEASIBILITY ANALYSIS	19
IV. SIMULATION OF THE NUCLEAR ROCKET ENGINE	33
The Constant-parameter System	34
The Reference Model	38
Adaptive Loop Design	42
Adaptive Loop Performance	44
V. PERFORMANCE OF MODEL REFERENCE ADAPTIVE CONTROL SYSTEM	47
Adaptive Control for a Varying Pressure Time Constant	47
Stability Considerations	64
VI. SUMMARY AND CONCLUSIONS	71
Summary of Results	71
Implications of Results	72
Suggestions for Further Study	73

TABLE OF CONTENTS (Continued)

	Page
APPENDICES	75
APPENDIX	
A. PARAMETER ADJUSTMENT ADAPTIVE CONTROL	76
B. EQUATION NORMALIZATION	85
Reactor Kinetics Equations	85
Temperature and Pressure Equations .	87
Reactivity and Controller Equations	88
C. ANALOG SIMULATION	89
Equipment	89
Analog Circuit Symbolism	89
Problem Scaling	92
Constant-parameter System Simulation	95
Reference Model Simulation	102
Adaptive Loop Simulation	102
D. DIGITAL CHECK	111
LIST OF REFERENCES	115
BIOGRAPHICAL SKETCH	118

LIST OF TABLES

TABLE	Page
1. ROCKET ENGINE CONSTANTS	14
2. INITIAL CONDITIONS	96
3. POTENTIOMETER SETTINGS FOR CONSTANT- PARAMETER ENGINE SIMULATION	100
4. POTENTIOMETER SETTINGS FOR MODEL SIMULATIONS	104
5. POTENTIOMETER SETTINGS FOR ADAPTIVE LOOP SIMULATION	107

LIST OF FIGURES

Figure		Page
1.	Parameter Adjustment Model Reference Adaptive Control System	7
2.	Schematic Diagram of a Nuclear Rocket Engine	11
3.	Block Diagram of Linearized Nuclear Rocket Engine	23
4.	Block Diagram of Simplified Linearized Nuclear Rocket Engine	26
5.	Effect of G_2 on Performance Index with $\tau_{mod} = 5^2$ Seconds	30
6.	Effect of G_2 on Performance Index with $\tau_{mod} = 10$ Seconds	31
7.	Response of Constant-parameter Nuclear Rocket Engine	37
8.	Responses of Models and Constant-parameter System	40
9.	Block Diagram of Adaptive Control System	43
10.	Nuclear Rocket Engine Response for τ_p (normal)	45
11.	Control Device Response for τ_p (normal)	46
12.	Nuclear Rocket Engine Response for 1.2 τ_p (normal)	49
13.	Control Device Response for 1.2 τ_p (normal)	50
14.	Nuclear Rocket Engine Response for 1.4 τ_p (normal)	51
15.	Control Device Response for 1.4 τ_p (normal)	52

LIST OF FIGURES (Continued)

Figure	Page
16. Nuclear Rocket Engine Response for 1.6 τ_p (normal)	53
17. Control Device Response for 1.6 τ_p (normal)	54
18. Nuclear Rocket Engine Response for 1.8 τ_p (normal)	55
19. Control Device Response for 1.8 τ_p (normal)	56
20. Nuclear Rocket Engine Response for 2 τ_p (normal)	57
21. Control Device Response for 2 τ_p (normal)	58
22. Nuclear Rocket Engine Response for 0.8 τ_p (normal)	60
23. Control Device Response for 0.8 τ_p (normal)	61
24. Constant-parameter and Adaptive Control Responses with τ_p Variation	62
25. Adaptation of Control Poison Proportional Control Gain	65
26. System Instability Due to Negative G_2^* .	67
27. Stability Boundary for Constant- parameter System	69
28. Generation of Response Error in Model Reference Adaptive Control System . .	77
29. Block Diagram of Second-order System with Adjustable Parameter in Feedforward Path	77

LIST OF FIGURES (Continued)

Figure		Page
30.	Block Diagram of Second-order System with Adaptive Control Loop	83
31.	Adaptive Response of Second-order System	84
32.	Symbols Used in Analog Computer Diagrams	90
33.	Analog Computer Diagram for Constant- parameter Nuclear Rocket Engine . . .	97
34.	Analog Computer Diagram for Constant- parameter Control System	98
35.	Analog Computer Diagrams for Reference Models	103
36.	Analog Computer Diagram for Adaptive Loop	105
37.	MIMIC Instructions	113

LIST OF SYMBOLS

C'	Normalized delayed neutron precursor density
G_1	Temperature integral control gain (sec^{-1})
G_2	Temperature proportional control gain
G_3	Pressure integral control gain (sec^{-1})
G_4	Pressure proportional control gain
l	Mean effective neutron lifetime (sec)
N'	Normalized neutron density
P	Core inlet pressure (psi)
P_d	Design core inlet pressure (psi)
P'	Normalized core inlet pressure
P'_x	Desired normalized core inlet pressure
s	Laplace transform variable (sec^{-1})
T	Maximum core surface temperature ($^{\circ}\text{R}$)
T_d	Design maximum core surface temperature ($^{\circ}\text{R}$)
T_r	Mean core temperature ($^{\circ}\text{R}$)
T'	Normalized maximum core surface temperature
T'_x	Desired maximum core surface temperature
V	Valve setting
V'	Normalized valve setting
V'_x	Desired normalized valve setting

LIST OF SYMBOLS (Continued)

α_t	Temperature coefficient of reactivity ($^{\circ}\text{R}^{-1}$)
α'_t	Normalized temperature coefficient of reactivity (dollars)
α_h	Propellant density coefficient of reactivity ($^{\circ}\text{R}/\text{psi}$)
α'_h	Normalized propellant density coefficient of reactivity (dollars)
β	Delayed neutron fraction
λ	Decay constant (sec^{-1})
ζ	Damping coefficient
ρ	Total reactivity
ρ'	Normalized total reactivity (dollars)
ρ_c	Control poison reactivity
ρ'_c	Normalized control poison reactivity (dollars)
ρ'_{cx}	Desired normalized control poison reactivity (dollars)
τ	Time constant (sec)
τ_p	Pressure time constant (sec)
τ_t	Thermal time constant (sec)
τ_p	Control poison time constant (sec)
τ_v	Valve time constant (sec)
ω_n	Natural frequency (sec^{-1})

Abstract of Thesis Presented to the Graduate Council
in Partial Fulfillment of the Requirements for the
Degree of Doctor of Philosophy

MODEL REFERENCE ADAPTIVE CONTROL OF A
NUCLEAR ROCKET ENGINE

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April, 1966

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Major Department: Nuclear Engineering Sciences

Parameter adjustment model reference adaptive control of a nuclear rocket engine is investigated. The engine considered is similar to the solid-core, 25,000 megawatt, million pound thrust model developed by Smith and Stenning. Proportional control gain for the control poison, i.e., rods or drums, is the parameter adaptively adjusted, and maximum core surface temperature is the adaptively controlled variable. Acceptable engine performance is achieved by using a second-order model with one finite zero.

Engine response to step commands is particularly sensitive to change in the system's pressure time constant. With constant-parameter control, doubling this time constant results in doubling the system's transition time from an idling condition to full power.

Adaptive control for the same condition achieves a transition time within 20 per cent of the desired, or "normal," response. Estimates of propellant savings with adaptive control are given, and dynamic stability of the system is discussed.

CHAPTER I

INTRODUCTION

The "case" for nuclear rocket propulsion has been firmly established (1, 2, 3).^{*} Arguments in favor of nuclear rocketry convincingly point up the dramatic increase in specific impulse and burnout velocity obtainable with nuclear-hydrogen rockets as compared with chemical rockets. There is general agreement that nuclear systems offer great promise for planetary missions requiring large payloads and high burnout velocities. With the assumption that development of nuclear rocket systems is essential to man's exploration of space, this study is directed to one of the problem areas associated with nuclear rocketry, namely, control of the rocket engine.

Mohler and Perry (4) observe that a nuclear rocket engine "requires a fairly sophisticated control system." They point out that rapid start-up (less than 1 minute) is necessary to conserve propellant and to reduce the

^{*}Underlined numbers in parentheses refer to entries in the List of References.

complexity of attitude control problems. One immediately recognizes this to be a much more difficult control problem than that encountered in operation of stationary power reactors. Investigators are using conventional feedback control techniques as well as modern automatic control theory in their attack upon the problem.

The control problem is complicated by a lack of agreement among investigators concerning certain key areas. For example, different opinions have been expressed as to the nature of temperature reactivity effects, a factor which significantly affects the design of any reactor control system. Stenning and Smith (5) contend that temperature reactivity is directly proportional to the square root of the core exit stagnation temperature, while Mohler and Perry (4) hold that temperature reactivity is directly proportional to this temperature. Weaver (6) observes, "both contentions could give fairly accurate results simply by choosing appropriate reactivity coefficients"; however, this is not a satisfying answer to the problem, and exhaustive analysis of the data available from rocket reactor tests is indicated. Perry (7) feels that temperature formulations in addition to those mentioned above will be considered before the matter is clarified. This all "adds up" to the fact that

nuclear rocket engines possess certain characteristics which are not at all well understood, even as design of the control system progresses.

Other considerations may also tend to complicate the design of an effective rocket engine control system. Are there effects of environment which tend to alter characteristics of the nuclear rocket engine? During a typical mission, for example, the engine will operate in low- or zero-gravity fields as well as in high-gravity fields. Effects of such environmental extremes upon flow and heat transfer have only recently received attention in depth (8). It may develop that gravitational effects can be expected to alter certain thermal characteristics of a nuclear heat-exchanger rocket engine.

It appears then that a control system must be designed for a vehicle with dynamics that are not thoroughly understood and with characteristics that may be dependent upon environmental conditions. Such a situation would seem to indicate the need for what is termed an "adaptive" control system.

The term "adaptive" first appeared in the control literature around eleven years ago, however, Draper (9) points out that adaptive features were employed in anti-aircraft fire-control systems as early as 1939. A concise definition of adaptivity has apparently been

difficult to formulate, and Truxal (10) suggests the following rather general idea:

A system is an adaptive control system if a means is provided to monitor, in addition to the state variable, its performance and/or process (internal and/or external) characteristics in order to accordingly modify the control action in an attempt to be an acceptably performing system.

Or, according to Whitaker (12):

An adaptive control system is defined to be a system which either by measurable adjustments of controllable parameters or by measurable and well-defined changes in dynamic characteristics can adapt to a changed environment, a changed character of input signals, or a changed system or component characteristic in such a manner that the specified desired performance is maintained.

Most of the adaptive control systems which have been studied can be included in the following three categories (11): 1) high gain systems, 2) model reference systems, and 3) optimum adaptive systems. The complexity of these systems increases with the order of the above listing. Donalson and Kishi (11) present rather comprehensive discussions of all three types of adaptive control, and it is not appropriate to include such a treatment here. Uhrig and Sage (13) have observed that the high gain systems do not appear particularly applicable to nuclear rocket control because the system would have to continually limit cycle at some frequency.

They further point out that the complexity of the optimum-adaptive schemes coupled with the fact that they are basically guidance techniques, and not control techniques, renders them also unattractive for this application. Such immediately obvious disadvantages do not exist in the case of model reference control systems.

As the name implies, the model reference control technique employs a model which embodies the design specifications for the system to be controlled. A command input is applied simultaneously to the reference model and to the basic control loop. The system output is compared with the reference model output, and a signal representing the error between actual and desired performance is made available. Requirement for parameter adjustment is determined by generating functions of the error, and parameter-variation commands are sent to the parameter adjustment devices in the control system. Thus the controller "adapts" in such a manner that the system output closely matches that of the model.

Model reference techniques have been classified in three categories (11): 1) parameter adjustment techniques, 2) parameter perturbation techniques, and 3) control signal synthesis techniques. The most popular technique, and the one used in this research, is the parameter

adjustment technique. Figure 1 depicts a model reference adaptive control system with provision for parameter adjustment. The basic control loop is enclosed in a dashed box. A response error signal is operated upon to bring about "optimum" control parameter variation, and as mentioned above, system output is closely matched with reference model output. Note that extent of parameter adjustment is determined solely by changes in characteristics of the controlled system since those of the model remain fixed.

Del Toro and Parker (14) have noted the following advantages, in addition to that of self-adaptability, of such a control system:

1. The adaptive loop functions independently of the main control loop. If the adaptive loop fails, operation is still possible.
2. The response error signal is immediately and continuously available, making disturbing test signals unnecessary.
3. Adaptive loops can be added to existing systems without appreciably altering the basic control loop.
4. Where several adaptive loops are required, several different models can be used.

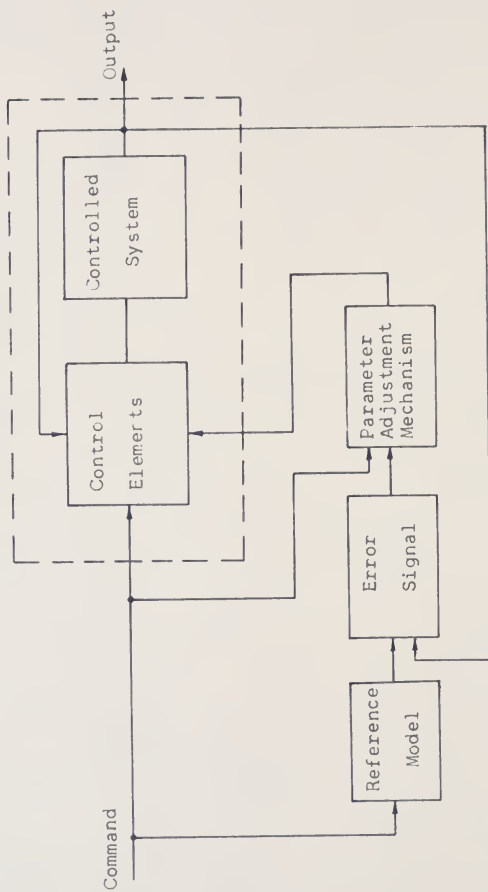


Figure 1. Parameter Adjustment Model Reference Adaptive Control System

5. Since noise in the input command is treated essentially in the same manner in the model as in the system, little, if any, deterioration in the optimizing process should occur.

This control technique is inherently nonlinear, time-varying, and dependent upon the time variation of the system parameters. The problem is usually too difficult for conventional analysis and so computer simulation is essential for design and analysis. The proliferation of model reference adaptive control schemes has not yet resulted in a generally accepted theory or procedure for design and analysis of the adaptive system. However, Osburn (15) has presented a design procedure which is quite attractive for many applications, and his approach was used in this investigation. A description of the derivation of his method along with a design example is included as Appendix A.

The objective of this research was to investigate the use of model reference parameter adjustment techniques in the control of a nuclear rocket engine. Chapters II and III contain a description of the nuclear rocket engine employed in the study, and an analysis of the feasibility of adaptive control for this application. Simulation of the engine is described in Chapter IV, and performance of the adaptive control

system is examined in Chapter V. Chapter VI includes a discussion of the results and suggestions for further study.

CHAPTER II

THE NUCLEAR ROCKET ENGINE

The nuclear rocket engine model employed in this study is based upon the work of Smith and Stenning (16, 17, 18, 19, 20). Their system has been extensively treated in the open literature, and exhibits the principal features of the solid-core reactor technology being developed in the United States for nuclear rockets (21).

Basic components of the engine are depicted in Figure 2. Liquid hydrogen, stored in low pressure tankage, is pumped to high pressure, and is used to regeneratively cool the nozzle and reflector. The propellant gas is further heated as it passes through the reactor core and is exhausted through the nozzle. A portion of the hot gas is bled off from the main stream and is used to power the turbopump, a feature giving rise to the name, "bleed turbine system." The bleed flow, and consequently the system pressure are controlled by the valve in front of the turbine.

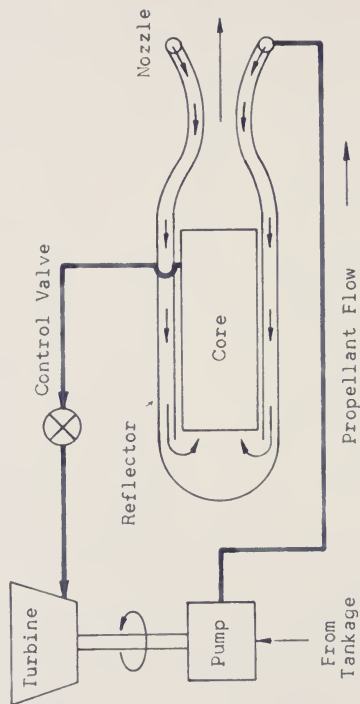


Figure 2. Schematic Diagram of a Nuclear Rocket Engine

The following equations describe the Smith-Stenning nuclear rocket engine in terms of time dependent normalized quantities. An account of the normalization procedure is contained in Appendix B.

$$\beta \frac{dN'}{dt} = \rho' N' - N' + C' \quad (2.1)$$

$$\frac{1}{\lambda} \frac{dC'}{dt} = N' - C' \quad (2.2)$$

$$\tau_t \frac{dT'}{dt} = N' - P'(T')^{1/2} \quad (2.3)$$

$$\tau_p \frac{dP'}{dt} = P' V'(T')^{1/2} - \frac{(P')^2}{(T')^{1/2}} \quad (2.4)$$

$$\rho' = \rho'_c + \alpha'_t (T')^{1/2} + \alpha'_h \frac{P'}{T'} \quad (2.5)$$

$$\rho_{cx}' = G_1 \int_{t_1}^{t_2} \left[(T'_x)^{1/2} - (T')^{1/2} \right] dt + \rho_{cx}'(0) \quad (2.6)$$

$$\tau_p \frac{d\rho'_c}{dt} = \rho_{cx}' - \rho'_c \quad (2.7)$$

$$V'_x = G_3 \int_{t_1}^{t_2} (P'_x - P) dt + G_4 (P'_x - P') + V'_x(0) \quad (2.8)$$

$$\tau_v \frac{dV'}{dt} = V'_x - V' \quad (2.9)$$

Equations 2.1 and 2.2 are the reactor kinetics equations.

Equation 2.3 gives the time rate of change of maximum core surface temperature, and equation 2.4 gives the time rate of change of the core inlet stagnation pressure. In the discussions which follow, "temperature" always refers to the maximum core surface temperature, and "pressure" always refers to the core inlet stagnation pressure. Reactivity is given by equation 2.5, and the engine's control system is represented by equations 2.6 through 2.9.

Table 1 contains important constants for the nuclear rocket engine. Note that the delayed neutron fraction is smaller than the value typically associated with a uranium-235 reactor. This reduced value was used by Smith and Stenning to account for diffusion of some precursors into coolant channels and their subsequent expulsion from the system before emission of delayed neutrons.

The above dynamic equations describe a nuclear rocket engine with thrust of 1,500,000 pounds, specific impulse of 640 seconds, and total power of 25,000 megawatts. These design specifications significantly exceed those of nuclear engines presently under development and will probably be first achieved through clustering of smaller engines. However, as indicated

TABLE 1
ROCKET ENGINE CONSTANTS

Design core surface temperature	5000 $^{\circ}\text{R}$
Design core inlet stagnation pressure	1500 psi
Thermal time constant	0.68 sec
Pressure time constant	2.50 sec
Control poison time constant	2.00 sec
Valve time constant	0.20 sec
Decay constant (one group)	0.10 sec^{-1}
Temperature reactivity coefficient	-10^{-6} $^{\circ}\text{R}^{-1}$
Hydrogen reactivity coefficient	0.3 $^{\circ}\text{R}/\text{psi}$
Neutron lifetime	10^{-4} sec
Delayed neutron fraction	0.005

above, the great amount of information in the open literature concerning the Smith-Stenning engine makes it attractive as the test vehicle for a control system study.

The design maximum core surface temperature at full power, or simply the design temperature, is 5000°R . Precise temperature control with small overshoot, i.e., less than 5 per cent, is required since the system operates near its limiting temperature in order to attain maximum specific impulse. Design core inlet stagnation pressure is 1500 psi. The terms "design temperature" and "design pressure" are used in the following discussions to denote conditions at full power.

It has been shown (17) that specific impulse is proportional to exit temperature and is independent of entrance pressure, while thrust is proportional to pressure, but independent of temperature. The feedback control system proposed by Smith and Stenning does not provide for independent control of specific impulse and thrust because coupling exists between propellant flow and reactivity. Their system is controlled by commanding step changes in both temperature and pressure, after which the engine proceeds to full power under its conventional feedback control system. The square root

of temperature used for temperature commands is obtained from measurements of propellant flow and pressure. Designers of the original engine considered this technique preferable to the use of slower thermocouple measurements.

The Smith-Stenning nuclear rocket engine model per se was not used in this research, but rather a modified version was deemed more suitable for this control system study. These modifications are now described.

Equation 2.5 indicates that temperature reactivity is directly proportional to the square root of temperature. As mentioned in the previous chapter, differences of opinion exist concerning this dependence. If one examines equation 2.5 and determines control poison reactivity at an idling condition where $T' = P' = 0.2$ and then compares this with control poison reactivity at full power where $T' = P' = 1.0$, it is noted that control poison movement is simply that of removal. If the $(T')^{1/2}$ term of equation 2.5 is replaced by T' , as in the Mohler-Perry (4) formulation, and if a conservative (low) value of the temperature reactivity is used, i.e., $-10^{-6} \text{ } ^\circ\text{R}^{-1}$, control poison movement is dramatically affected. During the transition

to full power, control poison is removed, but is finally returned to very nearly its original position. This means that the T' formulation, with its temperature coefficient, lessens the control effect of temperature and places more importance upon the control poison. Since this study was concerned with the practicability of a control technique, and the T' formulation appeared more rigorous, this modification was incorporated in the nuclear rocket engine model. Note that the term "control poison" is being used to avoid the limiting of control devices to conventional control rods. It is generally agreed that control drums in the reflector offer excellent control possibilities, and Bussard (3) has calculated that satisfactory response frequencies can be obtained with reasonable power requirements.

Another modification of the nuclear rocket engine model concerned proportional control of control poison. Smith and Stenning did not provide for such control in their model, however, in this study, proportional control of control poison was deemed practical.

The above modifications required that equations 2.5 and 2.6 be rewritten as follows:

$$\rho' = \rho'_c + \alpha'_t T' + \alpha'_h \frac{P'}{T'} \quad (2.10)$$

$$\begin{aligned} \rho_{cx}' = G_1 \int_{t_1}^{t_2} \left((T'_x)^{1/2} - (T')^{1/2} \right) dt \\ + G_2 \left((T'_x)^{1/2} - (T')^{1/2} \right) + \rho_{cx}'(0) \end{aligned} \quad (2.11)$$

One remaining modification had to do with turbine size. Smith and Stenning have presented results (19) of engine simulations requiring turbines capable of significantly greater flows than those encountered at design point operation, e.g., $V'_{\max} = 1.33$ and 3.0. They point out that although this increases speed of transition to full power, the heavier turbines lead to reduced burnout velocities. In this investigation, $V'_{\max}$ was limited to a value of 1.10. Chapter III is an adaptive control feasibility analysis for the modified nuclear rocket engine described above.

CHAPTER III

ADAPTIVE CONTROL FEASIBILITY ANALYSIS

According to Osburn (15),

Optimum performance of a model reference adaptive system is defined to be that which results when the system parameters are adjusted to produce a minimum of the performance index for the input or class of inputs for which performance is specified.

If this control technique is to be applied to a nuclear rocket system, then the designer must ensure the existence of a minimum of some selected performance index with respect to parameter variations. A preliminary analysis at this point will serve to indicate the feasibility for this particular application of model reference adaptive control.

The performance index (PI) used in this analysis is the integral squared response error, i.e.,

$$PI = \int_{t_1}^{t_2} e(t)^2 dt \quad (3.1)$$

where the response error $e(t)$ is formed by comparing system output with that of the reference model.

As is typically the case, development of needed transfer functions requires "linearization" of the rocket engine dynamic equations of Chapter II. Each time dependent quantity in these equations is assumed to be the sum of a steady-state value and a small deviation from that steady-state value. For example, N' is replaced by $N'_0 + \delta N'$. These substitutions are now performed to obtain the "linearized" nuclear rocket engine model. Equation 2.1 becomes

$$\begin{aligned} \tau_n(\dot{N}'_0 + \delta \dot{N}') &= (\rho'_0 + \delta \rho')(\dot{N}'_0 + \delta \dot{N}') \\ &\quad - (N'_0 + \delta N') + (C'_0 + \delta C') \end{aligned} \quad (3.2)$$

where the dot indicates differentiation with respect to time, and $\tau_n = 1/\beta$. Remembering that

$$\dot{N}'_0 = \rho'_0 \dot{N}'_0 - N'_0 + C'_0 \quad (3.3)$$

and neglecting second-order terms, equation 3.2 becomes

$$\tau_n \delta \dot{N}' = \rho'_0 \delta \dot{N}' + N'_0 \delta \rho' - \delta N' + \delta C' \quad (3.4)$$

In a similar fashion, equation 2.2 becomes

$$\tau_c \delta \dot{C}' = \delta N' - \delta C' \quad (3.5)$$

where $\tau_c = 1/\lambda$. Linearizing equation 2.3 gives

$$\begin{aligned} \tau_t(T'_0 + \delta T') &= (N'_0 + \delta N') \\ &\quad - (P'_0 + \delta P')(T'_0 + \delta T')^{1/2} \end{aligned} \quad (3.6)$$

and noting that

$$(T'_0 + \delta T')^{1/2} \approx (T'_0)^{1/2} + \frac{\delta T'}{2(T'_0)^{1/2}} \quad (3.7)$$

equation 3.6 becomes

$$\tau_t \dot{\delta T'} = \delta N' - (T'_0)^{1/2} \delta P' - \frac{P'_0}{2(T'_0)^{1/2}} \delta T' \quad (3.8)$$

Similarly, by using

$$(T'_0 + \delta T')^{-1/2} \approx (T'_0)^{-1/2} \left(1 - \frac{\delta T'}{2(T'_0)}\right) \quad (3.9)$$

and by dropping steady-state terms, equation 2.4 becomes

$$\begin{aligned} \tau_p \dot{\delta P'} &= \frac{P'_0 V'_0}{2(T'_0)^{1/2}} \delta T' + V'_0 (T'_0)^{1/2} \delta P' \\ &\quad + P' (T'_0)^{1/2} \delta V' + \frac{(P'_0)^2}{2(T'_0)^{3/2}} \delta T' - \frac{2P'_0}{(T'_0)^{1/2}} \delta P' \end{aligned} \quad (3.10)$$

The reactivity equation, i.e., equation 2.10, with the temperature and hydrogen coefficients in expanded form, is written as

$$\rho' = \rho'_c + \frac{\alpha_t T_d}{\beta} T' + \frac{\alpha_h P_d}{\beta T_d} \frac{P'}{T'} \quad (3.11)$$

Linearizing equation 3.11 results in

$$\delta \rho' = \delta \rho'_c + A_t \delta T' + A_h \delta P' \quad (3.12)$$

where

$$A_t = \frac{\alpha_t T_d}{\beta} - \frac{\alpha_h P_d}{\beta T_d} \frac{P_0'}{(T_0')^2} \quad (3.13)$$

and

$$A_h = \frac{\alpha_h}{\beta} \frac{P_d}{T_d} \frac{1}{T_0'} \quad (3.14)$$

The controller equations are treated similarly, and equations 2.7, 2.8, 2.9, and 2.11 become, respectively,

$$\begin{aligned} \delta \rho'_{cx} = & \frac{G_1}{2} \int_{t_1}^{t_2} \left(\frac{\delta T'_x}{(T'_{x0})^{1/2}} - \frac{\delta T'}{(T_0')^{1/2}} \right) dt \\ & + \frac{G_2}{2} \left(\frac{\delta T'_x}{(T'_{x0})^{1/2}} - \frac{\delta T'}{(T_0')^{1/2}} \right) \end{aligned} \quad (3.15)$$

$$\tau_p \dot{\delta \rho'_c} = \delta \rho'_{cx} - \delta \rho'_c \quad (3.16)$$

$$\delta v'_x = G_3 \int_{t_1}^{t_2} (\delta P'_x - \delta P') dt + G_4 (\delta P'_x - \delta P') \quad (3.17)$$

$$\tau_v \dot{\delta v'} = \delta v'_x - \delta v' \quad (3.18)$$

A block diagram of the linearized nuclear rocket engine, in terms of the Laplace transform variable s , is contained in Figure 3. Note that the engine is essentially a reactor-temperature loop coupled with a turbomachinery-pressure loop.

The set of linearized equations does not easily lend itself to analysis, and the following simplifying assumptions are employed. The prompt neutron lifetime is set equal to zero, an assumption which is valid for reactivities less than 0.6β (22). Further, effects of delayed neutrons are neglected, a conservative assumption, and no time lag is associated with the control poison. The reasonableness of these assumptions was supported by analog computer simulation. Finally, the valve opening is held constant, and temperature is the controlled variable.

With these assumptions and restrictions, the linearized model equations for the feasibility study become

$$N'_0 \delta \rho' = \delta N' \quad (3.19)$$

$$\tau_t \dot{\delta T} = \delta N' - (T'_0)^{1/2} \delta P' - \frac{P'_0}{2(T'_0)^{1/2}} \delta T' \quad (3.20)$$

$$\begin{aligned} \tau_p \dot{\delta P} = & \left(\frac{P'_0 V'_0}{2(T'_0)^{1/2}} + \frac{(P'_0)^2}{2(T'_0)^{3/2}} \right) \delta T' \\ & + \left(V'_0 (T'_0)^{1/2} - \frac{2P'_0}{(T'_0)^{1/2}} \right) \delta P' \end{aligned} \quad (3.21)$$

$$\delta \rho' = \delta \rho'_c + A_t \delta T' + A_h \delta P' \quad (3.22)$$

$$\delta \rho'_{cx} = \delta \rho'_c = \frac{G_1}{2} \int_{t_1}^{t_2} \left(\frac{\delta T'_x - \delta T'}{(T'_0)^{1/2}} \right) dt + \frac{G_2}{2} \left(\frac{\delta T'_x - \delta T'}{(T'_0)^{1/2}} \right) \quad (3.23)$$

Figure 4 is a block diagram of the simplified linearized rocket engine model.

Determination of the performance index, i.e., the integral squared response error, requires that a reference model be chosen. For this analysis, a first-order model with transfer function (TF_{mod}) equal to $1/(\tau s + 1)$ was selected to represent the desired temperature response, and a unit step was used as the command input. This means that the transformed response error is given by

$$e(s) = \frac{1}{s} (TF_{sys} - TF_{mod}) \quad (3.24)$$

The block diagram of Figure 4 can be used to determine the transfer function of the simplified linearized rocket engine system (TF_{sys}), where

$$TF_{sys} = \frac{\delta T'(s)}{\delta T'_x(s)} \quad (3.25)$$

The system transfer function is given by

$$TF_{sys} = F \frac{G_2 s^2 + (G_1 + G_2 R_2) s + R_2 G_1}{s^3 + X s^2 + Y s + F G_1 R_2} \quad (3.26)$$

where

$$F = \frac{N'_0}{2 \tau_t (T'_0)^{1/2}} \quad (3.27)$$

$$X = R_1 + R_2 + F G_2 + A_t \frac{N'_0}{\tau_t} \quad (3.28)$$

$$Y = R_1 R_2 - A_h \frac{N'_0}{\tau_t} E + F (G_1 + R_2 G_2) + A_t \frac{N'_0 R_2}{\tau_t} + \frac{(T'_0)^{1/2}}{\tau_t} E \quad (3.29)$$

$$R_1 = \frac{P'_0}{2 \tau_t (T'_0)^{1/2}} \quad (3.30)$$

$$R_2 = \frac{P'_0}{\tau_p (T'_0)^{3/2}} \quad (3.31)$$

$$E = \frac{(P'_0)^2}{\tau_p (T'_0)^{3/2}} \quad (3.32)$$

Substitution of equation 3.26 in equation 3.24

gives

$$e(s) = \left[(F G_2 - 1) s^2 + (F [G_1 \tau + G_2 R_2 \tau + G_2] - X) s + F (G_1 + G_2 R_2 + G_1 R_2 \tau) - Y \right] / \left[\tau s^4 + (\tau X + 1) s^3 + (X + \tau Y) s^2 + (Y + F R_2 G_1 \tau) s + F \right] \quad (3.33)$$

The performance index, as given by equation 3.1, can now be determined as a function of the proportional control gain G_2 by using Parseval's theorem (23). This represents a convenient way of expressing the integral squared value of a time function in terms of its transform, i.e., if

$$I = \int_{-\infty}^{\infty} dt X^2(t) \quad (3.34)$$

then

$$I = \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} ds X(-s)X(s) \quad (3.35)$$

Equation 3.35 can be written as

$$I_n = \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} ds \frac{c(s)c(-s)}{d(s)d(-s)}$$

where

$$c(s) = c_{n-1}s^{n-1} + \cdots + c_0 \quad (3.36)$$

and

$$d(s) = d_n s^n + \cdots + d_0 \quad (3.37)$$

Fortunately, values of this integral have been tabulated, and for the simplified rocket engine, where $n = 4$, the integral squared response error is given by

$$I = \frac{c_3^2(-d_0^2d_3 + d_0d_1d_2) + (c_2^2 - 2c_1c_3)d_0d_1d_4}{2d_0d_4(-d_0d_3^2 - d_1^2d_4 + d_1d_2d_3)} + \frac{(c_1^2 - 2c_0c_2)d_0d_3d_4 + c_0^2(-d_1d_4^2 + d_2d_3d_4)}{2d_0d_4(-d_0d_3^2 - d_1^2d_4 + d_1d_2d_3)} \quad (3.38)$$

Values of the performance index were calculated as a function of the proportional gain G_2 and are presented in Figures 5 and 6. The number associated with each performance index curve identifies the steady-state condition of the engine when it was perturbed, i.e., $T'_0 = P'_0 = 0.1$, $T'_0 = P'_0 = 0.2$, etc.

Figure 5 represents the response error for a reference model with time constant equal to 5 seconds, while the model for Figure 6 has a 10-second time constant. In both cases, well defined minima were obtained for nearly all conditions, indicating that adaptive control of temperature with a variable gain in the proportional controller for control poison is feasible. These curves point up the fact that the engine's response is dependent upon power level. This is, in effect, another argument for adaptive control of nuclear rocket engines since this can be likened to a system characteristic that changes with environment.

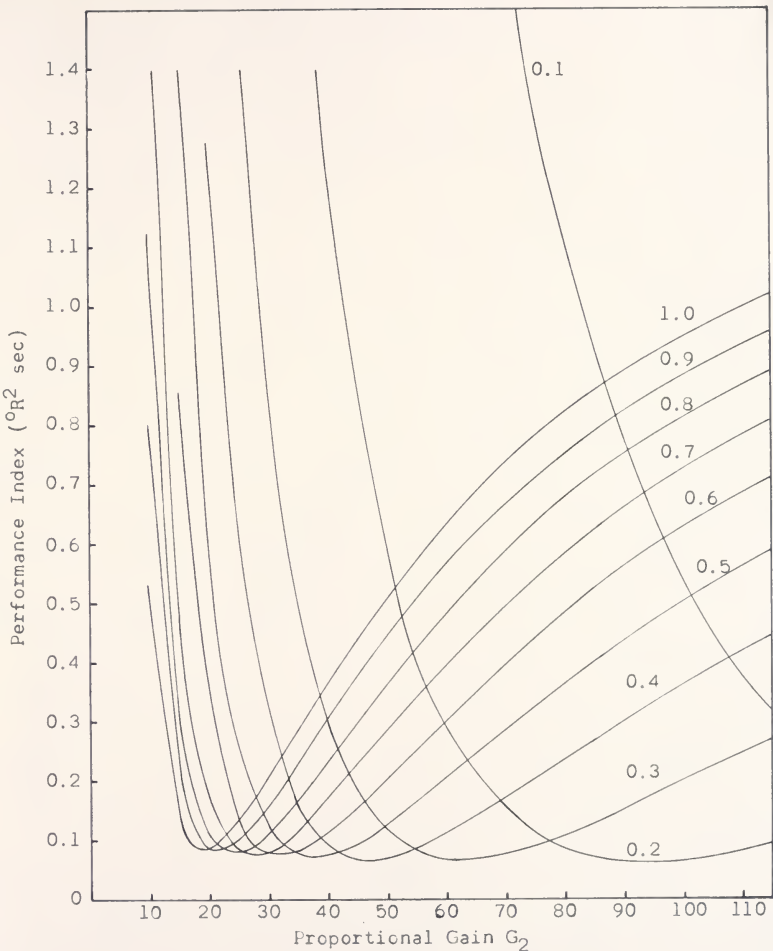


Figure 5. Effect of G_2 on Performance Index with $\tau_{\text{mod}} = 5$ Seconds

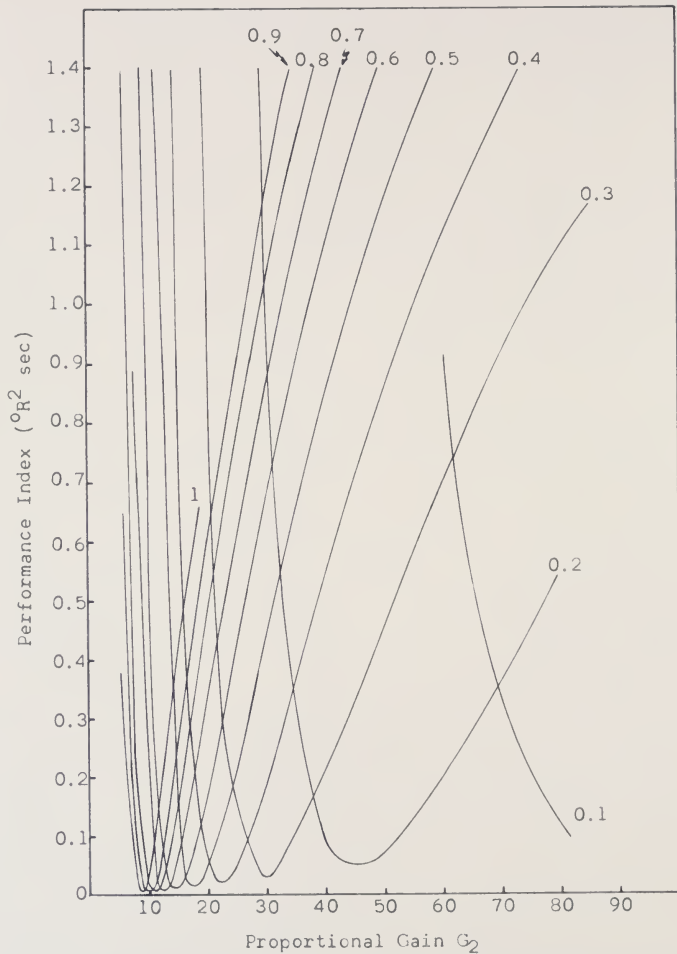


Figure 6. Effect of G_2 on Performance Index with $\tau_{\text{mod}} = 10 \text{ Seconds}$

The results of this preliminary analysis indicated that adaptive control of the simplified nuclear rocket engine was feasible, and simulation of the complete system followed. This simulation is described in Chapter IV.

CHAPTER IV

SIMULATION OF THE NUCLEAR ROCKET ENGINE

Simulation of the nuclear rocket engine and its control system was performed with an analog computer. A description of the analog computer equipment used in this work is presented in Appendix C. Block diagrams as well as detailed computer wiring diagrams are presented as the simulation is described. The investigation proceeded in the following manner. A constant parameter nuclear rocket engine was simulated for the purpose of determining a desired, or "normal," performance with a conventional feedback control system. A reference model closely approximating this desired behavior was then chosen, and an adaptive loop including the parameter adjustment mechanism was designed and simulated. Finally, performance of the model reference adaptively controlled nuclear rocket engine was studied as a system parameter was varied.

The Constant-parameter System

The system equations of Chapter II, including the constants of Table 1, are repeated here for convenience.

$$\dot{N}' = 50(\rho' N' - N' + C') \quad (4.1)$$

$$\dot{C}' = 0.1(N' - C') \quad (4.2)$$

$$\dot{T}' = 1.471 \left(N' - P'(T')^{1/2} \right) \quad (4.3)$$

$$\dot{P}' = 0.4 \left(P' V'(T')^{1/2} - \frac{(P')^2}{(T')^{1/2}} \right) \quad (4.4)$$

$$\rho' = \rho'_c - T' + \frac{18P'}{T'} \quad (4.5)$$

$$\begin{aligned} \rho'_{cx} = G_1 \int_0^t \left[(T'_x)^{1/2} - (T')^{1/2} \right] dt \\ + G_2 \left[(T'_x)^{1/2} - (T')^{1/2} \right] + \rho'_{cx}(0) \end{aligned} \quad (4.6)$$

$$\dot{\rho}'_c = 0.5(\rho'_{cx} - \rho'_c) \quad (4.7)$$

$$V'_x = G_3 \int_0^t (P'_x - P') dt + G_4(P'_x - P') + V'_x(0) \quad (4.8)$$

$$\dot{V}' = 5(V'_x - V') \quad (4.9)$$

Simulation of a system on an analog computer requires amplitude scaling of the system equations. This scaling is necessary because an analog computer establishes mathematical relations between machine voltages

which represent problem variables. These voltages must be kept within specified limits (± 100 volts for the equipment used in this work) in order to prevent overloading of machine components. Such overloading, or saturation, leads to incorrect computation. Details of the amplitude scaling for this problem are included in Appendix C.

Amplitude scaling of equations 4.1-9 results in the following set, where the bars indicate that quantities are now expressed in volts, i.e., the equations are written in terms of machine variables.

$$\dot{\bar{N}} = 100\bar{Q} - 50\bar{N} + 50\bar{C} \quad (4.10)$$

$$\dot{\bar{C}} = 0.1(\bar{N} - \bar{C}) \quad (4.11)$$

$$\dot{\bar{T}} = 14.71\bar{N} - 4.16\bar{R} \quad (4.12)$$

$$\dot{\bar{P}} = 1.7\bar{S} - 11.32\bar{Z} \quad (4.13)$$

$$\bar{\rho} = 10\bar{\rho}_c - \bar{T} + 180\bar{Y} \quad (4.14)$$

$$\begin{aligned} \bar{\rho}_{cx} = & \frac{G_1}{2^{1/2}} \int_0^t \left| (\bar{T}_x)^{1/2} - (\bar{T})^{1/2} \right| dt \\ & + \frac{G_2}{2^{1/2}} \left| (\bar{T}_x)^{1/2} - (\bar{T})^{1/2} \right| + \bar{\rho}_{cx}(0) \end{aligned} \quad (4.15)$$

$$\dot{\bar{\rho}}_c = 0.5(\bar{\rho}_{cx} - \bar{\rho}_c) \quad (4.16)$$

$$\bar{V}_x = 1.33G_3 \int_0^t (\bar{P}_x - \bar{P})dt + 1.33G_4(\bar{P}_x - \bar{P}) + \bar{V}_x(0) \quad (4.17)$$

$$\dot{\bar{V}} = 5(\bar{V}_x - \bar{V}) \quad (4.18)$$

where

$$\bar{Q} = \bar{P} \frac{\bar{N}}{100} \quad (4.19)$$

$$\bar{R} = \bar{P} \frac{(\bar{T})^{1/2}}{10} \quad (4.20)$$

$$\bar{Y} = 10 \frac{\bar{P}}{\bar{T}} \quad (4.21)$$

$$\bar{S} = \bar{V} \frac{\bar{R}}{100} \quad (4.22)$$

$$\bar{Z} = \bar{R} \frac{\bar{Y}}{100} \quad (4.23)$$

The rationale underlying the grouping of quantities as expressed in equations 4.19-23 is discussed in Appendix C.

The above equations were used for simulation of the constant-parameter nuclear rocket engine. Figure 7 depicts response of the system to a step command calling for transition from an idling condition ($T = 1000^\circ R$, $P = 275$ psi) to the full power condition ($T = 5000^\circ R$, $P = 1500$ psi). Controller gain settings are listed in

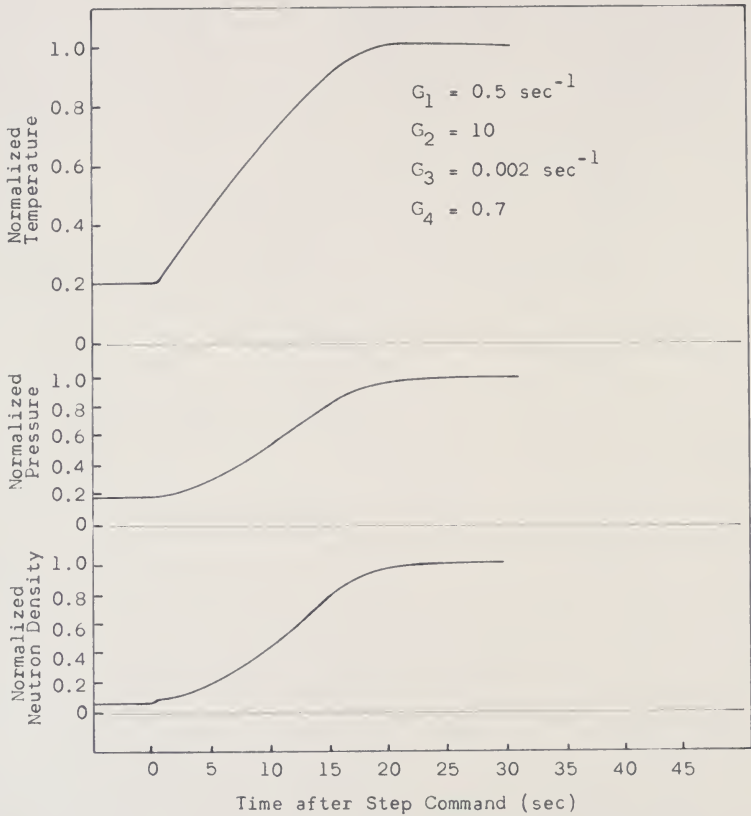


Figure 7. Response of Constant-parameter Nuclear Rocket Engine

the figure. Figure 7 does not indicate the existence of intolerable temperature transients, and the overshoot is less than an acceptable 5 per cent. This response was chosen as the "best" attainable with this design and is hereafter referred to as the system's "normal" response.

The Reference Model

The normal performance depicted in Figure 7 is the performance that the model reference adaptive control system is to maintain despite changes in characteristics of the nuclear rocket engine. This means that a reference model must be developed with a response closely resembling this normal response.

An important criterion for the reference model is that it be as simple as possible, i.e., that it require a minimum of control equipment above and beyond the equipment requirements of the conventional feedback control system. The analysis of Chapter III, together with the system's simplified transfer function, suggests that a reasonable first choice for the reference model would be one with the following transfer function:

$$TF_{\text{mod}} = \frac{1}{\tau_s + 1} \quad (4.24)$$

The response of such a first-order model with a time constant of 10 seconds is contained in Figure 8 along with the normal system response and the responses of two other models which are discussed below. The responses are for a step command corresponding to the command calling for transition from an idling condition to the full power condition.

It is obvious that the "tailing off" of the first-order model's response as it approaches the full power condition would result in excessive transition times if used as the reference model in an adaptive control system. If attempts are made to reduce this "rise time," severe temperature transients and their attendant thermal shock problems would undoubtedly arise. These observations cause one to consider a more complicated, yet simple, reference model. This is a second-order model with transfer function as follows:

$$TF_{\text{mod}} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (4.25)$$

Figure 8 contains the response of this model for

$\zeta = 0.7$, $\omega_n = 0.179 \text{ sec}^{-1}$. This model is an improvement over the first-order model in approximating the normal rocket engine response. Overshoot and time for the idle to full power transition are acceptable. However, note that at the instant of command, i.e., $t = 0$,

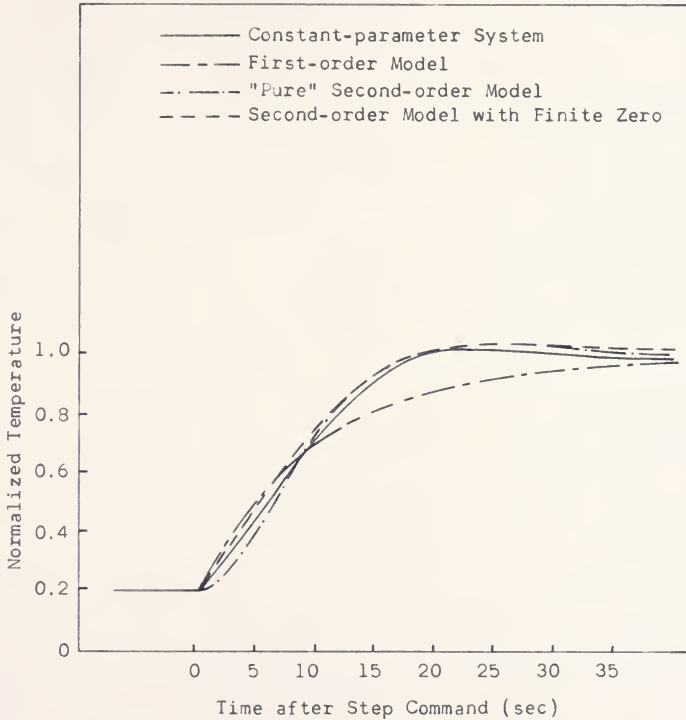


Figure 8. Responses of Models and Constant-parameter System

the slope of the model's transient response is zero. This means that at the instant of command, and for a short time thereafter, such a reference model would demand a degradation of the nuclear rocket engine's normal response. The model would later counteract this action; however, this situation does not appear attractive.

It appears that a second-order reference model might suffice for adaptive control of the rocket engine, provided that its initial response can be made to better approximate the normal system response. This is accomplished by adding a zero to the transfer function of the second-order model, i.e.,

$$TF_{\text{mod}} = \frac{(\omega_n^2/a)(s + a)}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (4.26)$$

Figure 8 contains the response of such a model with $\zeta = 0.74$, $\omega_n = 0.16 \text{ sec}^{-1}$, and $\omega_n^2/a = 0.049$. It is observed that this second-order model with a zero in its transfer function very closely approximates the rocket engine's normal response during the early portion of the idle to full power transition. This is desirable and means that the system's normal response will not be affected in this important phase of the transition by the adaptive controller unless, of course, changed

characteristics have altered the nuclear rocket engine's transient response. With the realization that a perfect system-model match can be attained only when the two transfer functions are identical, the second-order model with a zero was chosen as the reference model for the nuclear rocket engine. Detailed analog computer diagrams for the three reference models discussed above are contained in Appendix C.

Adaptive Loop Design

With a reference model selected, it is now possible to design the adaptive loop, which includes the reference model, weighting function filter, and parameter adjustment mechanism. These components are discussed in detail both in Chapter III and in Appendix A. The example contained in Appendix A should be of help in following the design of the adaptive loop for the nuclear rocket engine control system. A block diagram of the adaptive loop is contained in Figure 9. Design of the loop closely follows Osburn's procedure. The weighting function filter has the reference model transfer function. Signal input to the adjustable parameter is passed through this filter and is multiplied by the error signal resulting from subtraction of the model output from that of the system. This product

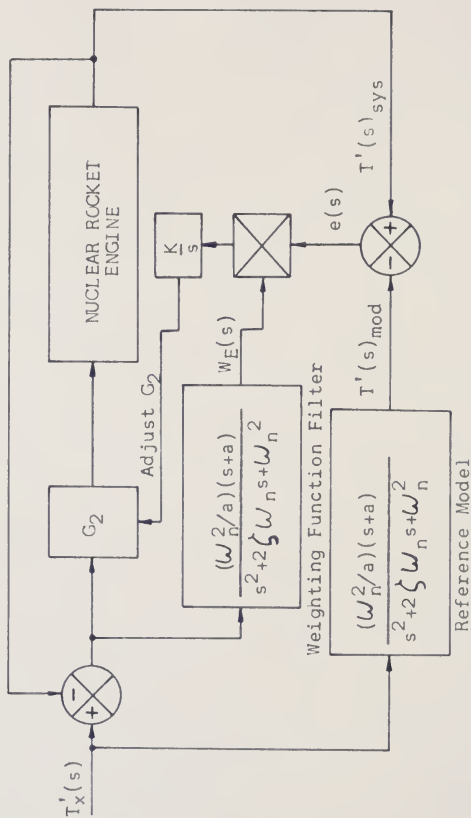


Figure 9. Block Diagram of Adaptive Control System

is integrated and serves to vary the adjustable gain automatically. The analog computer diagram for the adaptive loop is contained in Appendix C.

Adaptive Loop Performance

It is now possible to study the effect on normal system performance of addition of the adaptive control loop. Figures 10 and 11 depict response of the nuclear rocket engine going from idle to full power with and without the adaptive controller. Note that there is slight upgrading of the temperature response, but, in general, the normal response of the system is relatively unchanged by addition of the adaptive control loop. With the adaptive controller operative, it is now appropriate to evaluate the control system's effectiveness in maintaining normal performance of the nuclear rocket engine as an important engine characteristic is varied. This evaluation is described in the following chapter.

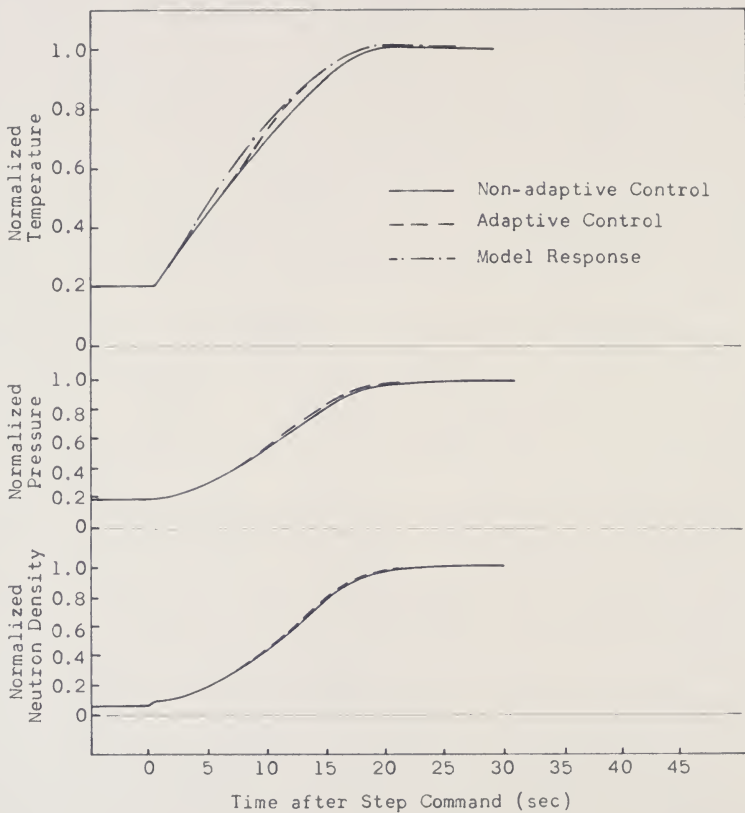


Figure 10. Nuclear Rocket Engine Response for τ_p (normal)

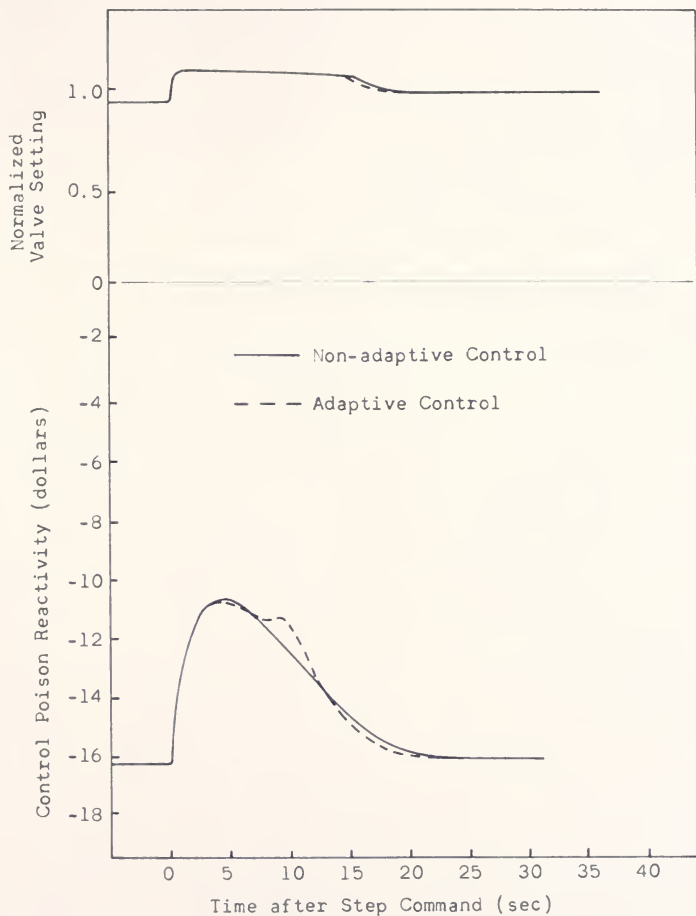


Figure 11. Control Device Response for τ_p (normal)

CHAPTER V

PERFORMANCE OF MODEL REFERENCE ADAPTIVE CONTROL SYSTEM

Study of the applicability of model reference techniques to control of the nuclear rocket engine has developed in the following manner. An engine including a conventional feedback control system was simulated on the analog computer. A "normal," or desirable, performance was established, and this performance is to be achieved in the face of changed rocket engine characteristics by means of a model reference adaptive control system. A reference model matching this normal performance was chosen and utilized in the design and simulation of an adaptive control loop. It now remains to be seen whether or not the adaptive control system is effective in achieving the specified performance.

Adaptive Control for a Varying Pressure Time Constant

The nuclear rocket engine model used in this study is particularly sensitive to changes in the pressure time constant, and it is this parameter which was varied

to evaluate the effectiveness of the adaptive control system. The parameter was varied from its expected, i.e., normal, value, to two times this value.

Figures 12 through 21 depict response of the nuclear rocket engine for various values of the pressure time constant. Response of the constant-parameter system, as well as the reference model response, is presented along with the adaptively controlled response for each case. In addition, pressure, neutron density, valve position, and control poison reactivity are shown for each type of control.

Note that the constant-parameter, i.e., non-adaptive, system's response deteriorates and becomes more sluggish as the time constant increases. On the other hand, the adaptively controlled system "sees" this deterioration and acts to achieve the desired performance. Although temperature is the controlled variable, the coupled system variables reflect this same behavior. It is interesting to observe the control poison reactivity as a function of time after the step command to proceed from the idling condition to full power. In each case, more positive reactivity is introduced to compensate for the sluggish turbomachinery.

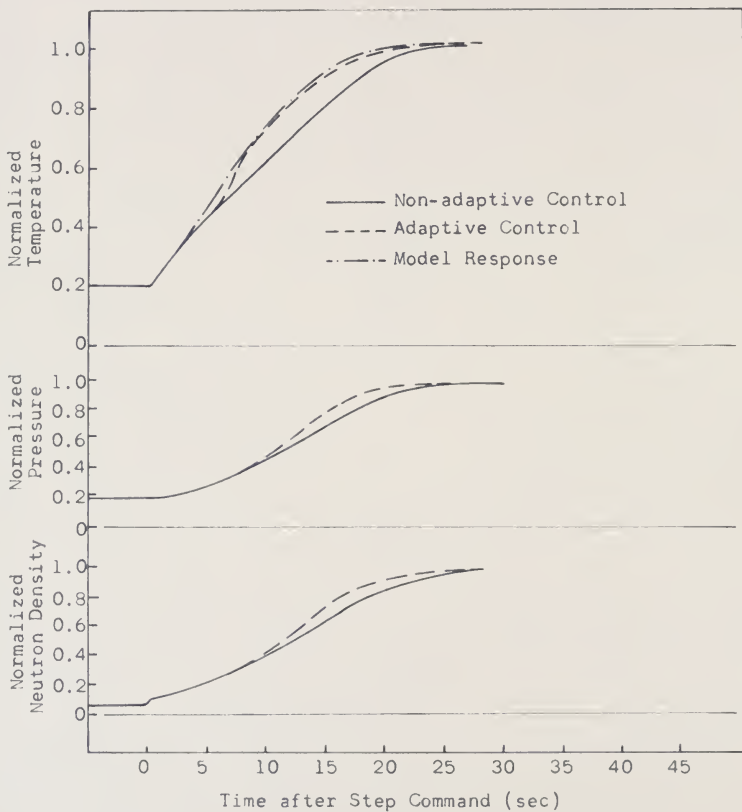


Figure 12. Nuclear Rocket Engine Response for 1.2 τ_p (normal)

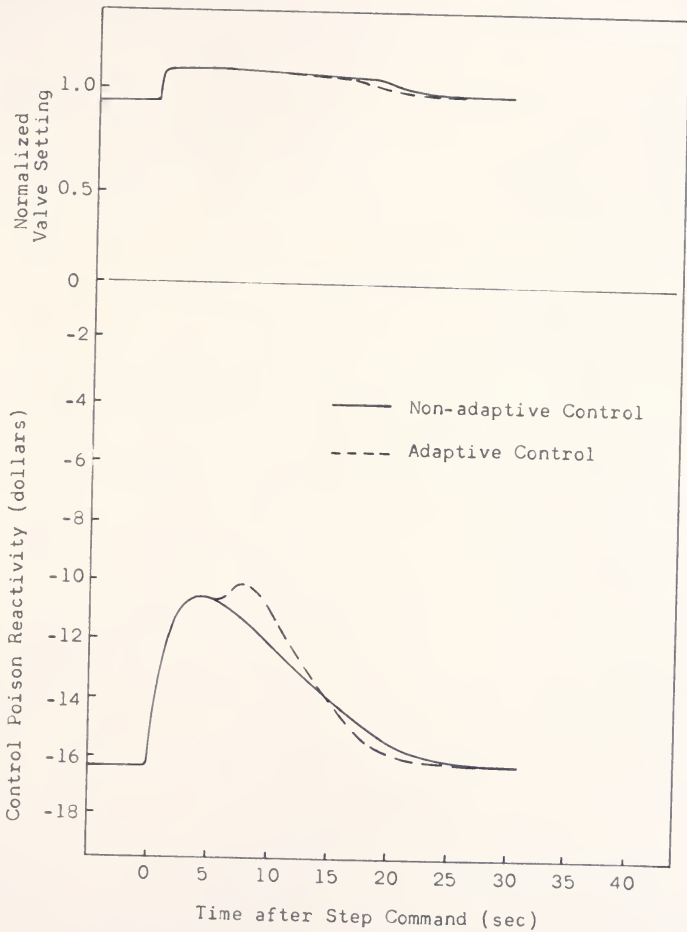


Figure 13. Control Device Response for $1.2 \tau_p$ (normal)

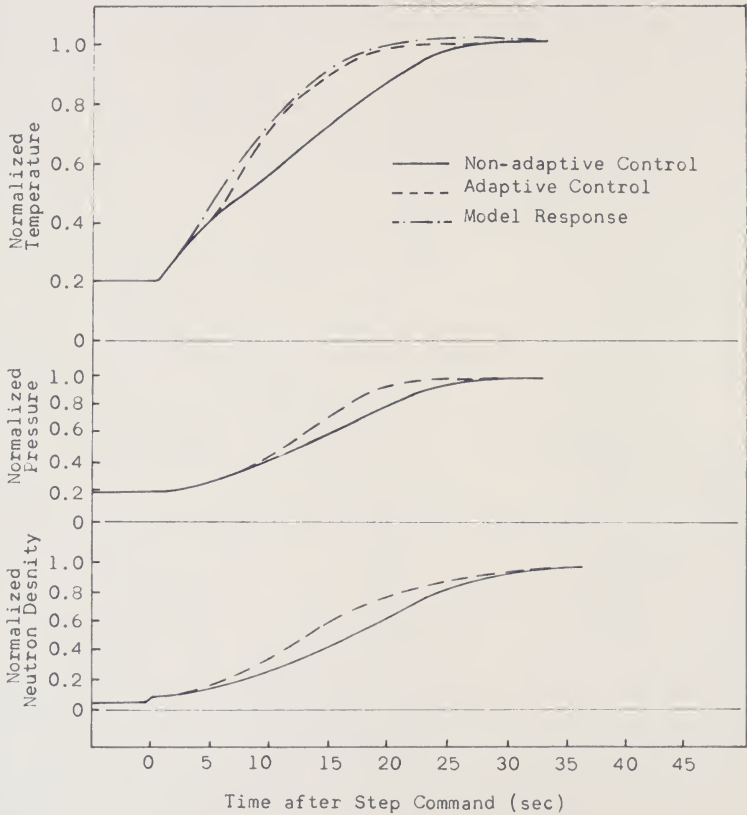


Figure 14. Nuclear Rocket Engine Response for $1.4 \tau_p$ (normal)

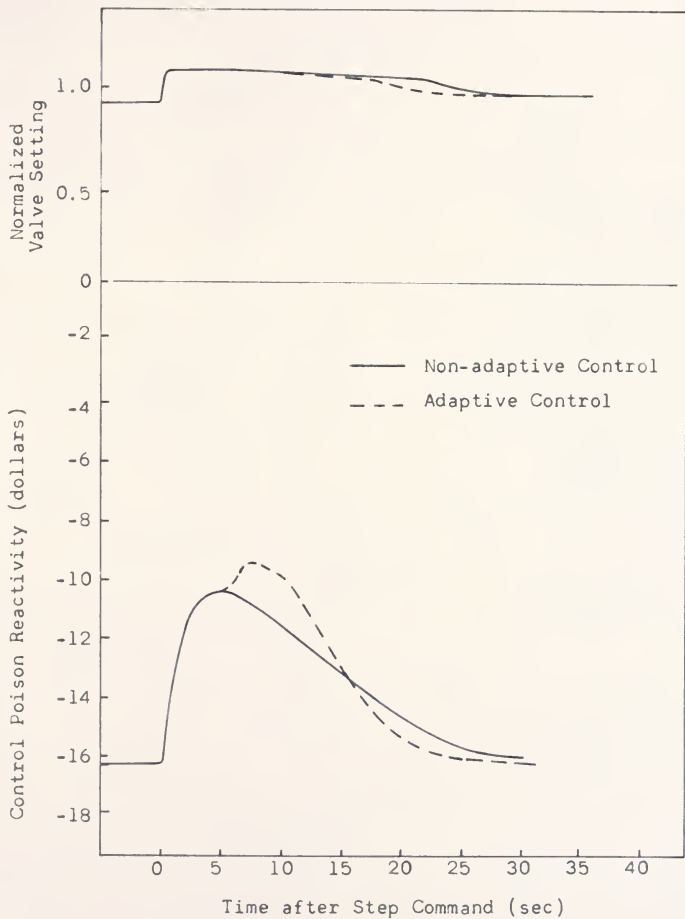


Figure 15. Control Device Response for $1.4 \tau_p$ (normal)

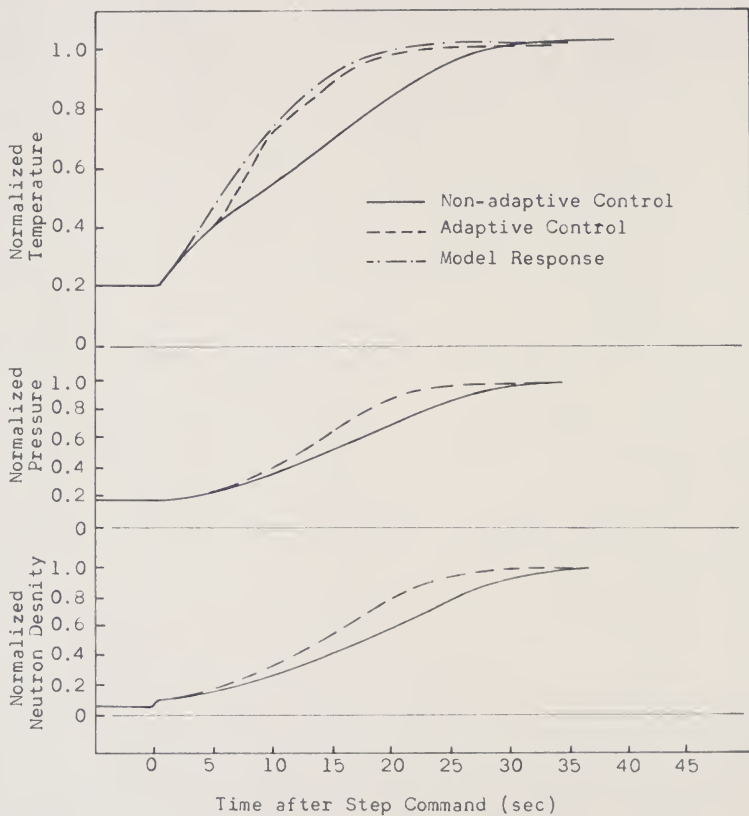


Figure 16. Nuclear Rocket Engine Response for $1.6 \tau_p$ (normal)

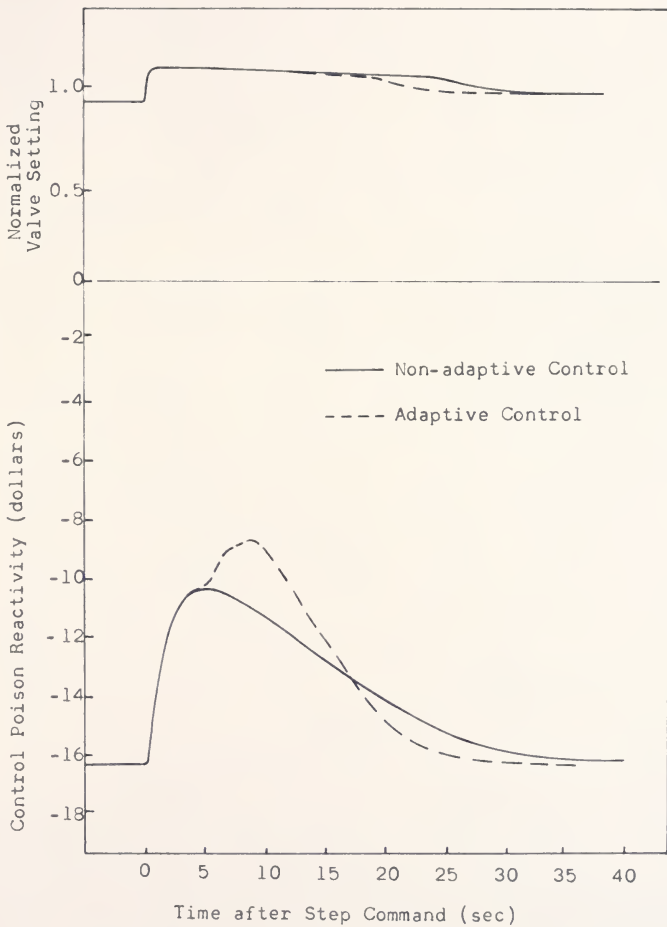


Figure 17. Control Device Response for $1.6 \tau_p$ (normal)

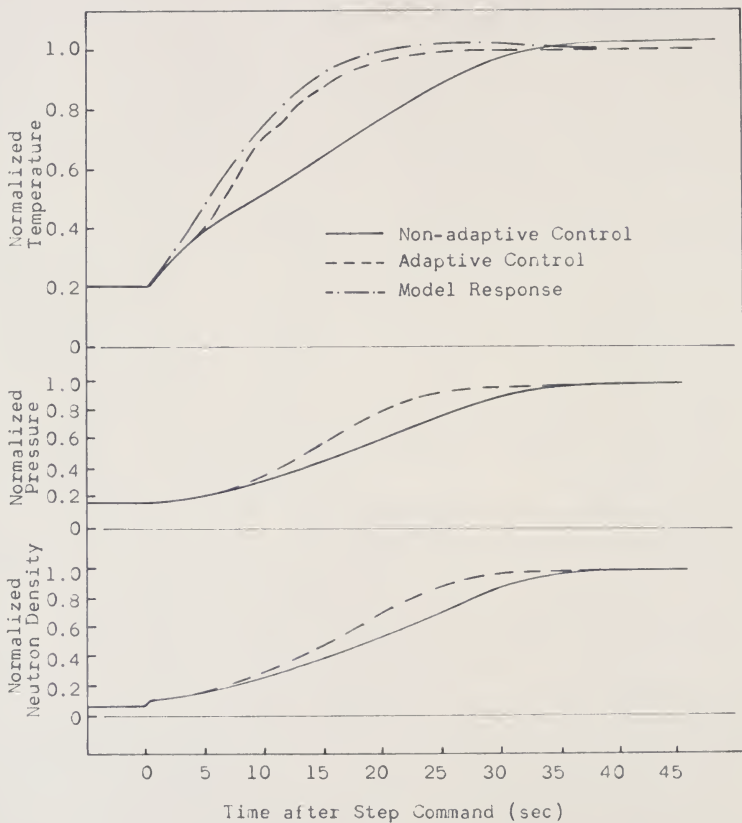


Figure 18. Nuclear Rocket Engine Response for $1.8 \tau_p$ (normal)

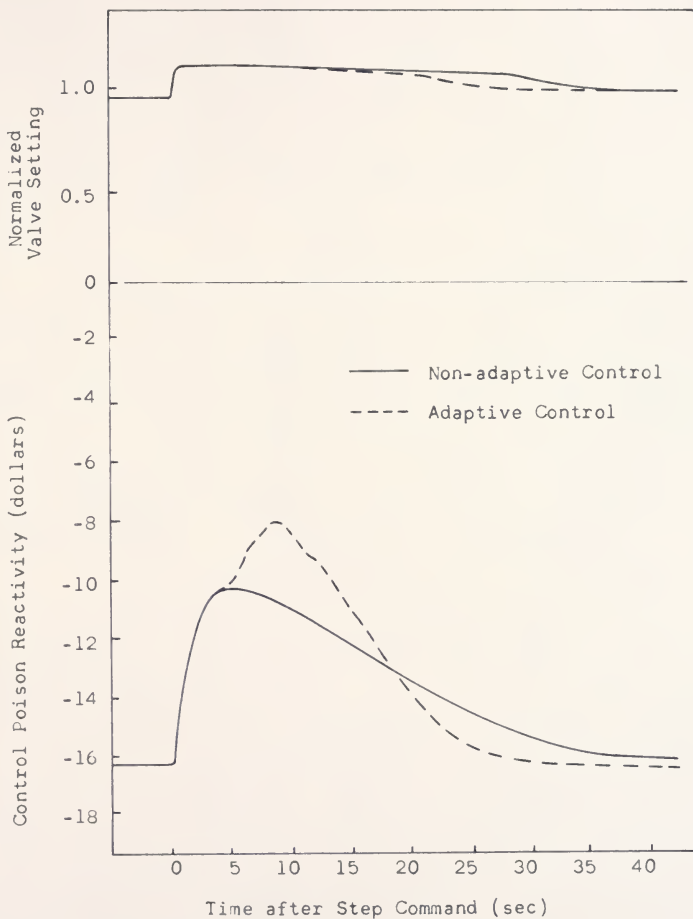


Figure 19. Control Device Response for $1.8 \tau_p$ (normal)

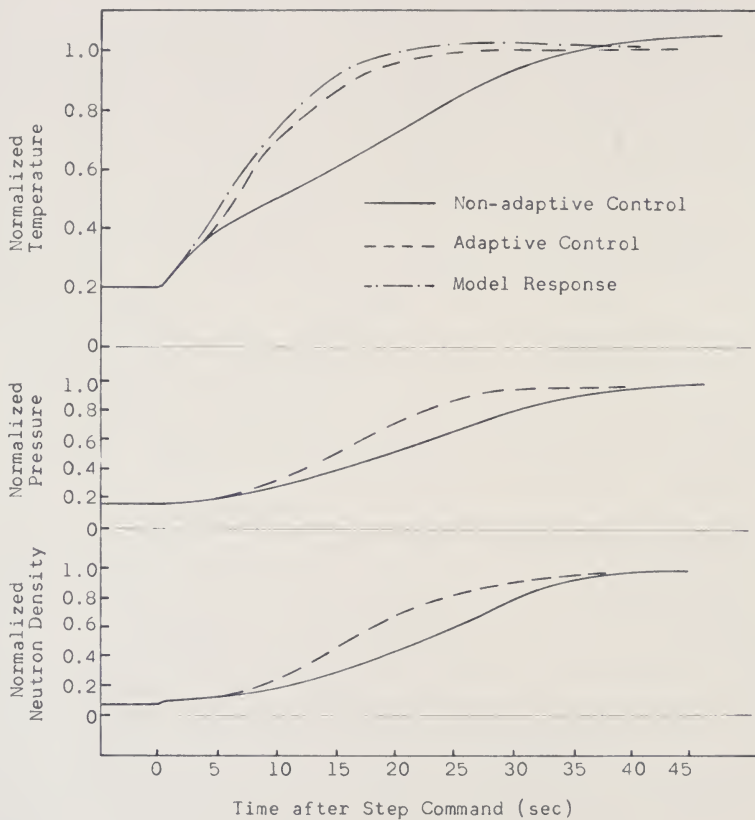


Figure 20. Nuclear Rocket Engine Response for $2 T_p$ (normal)

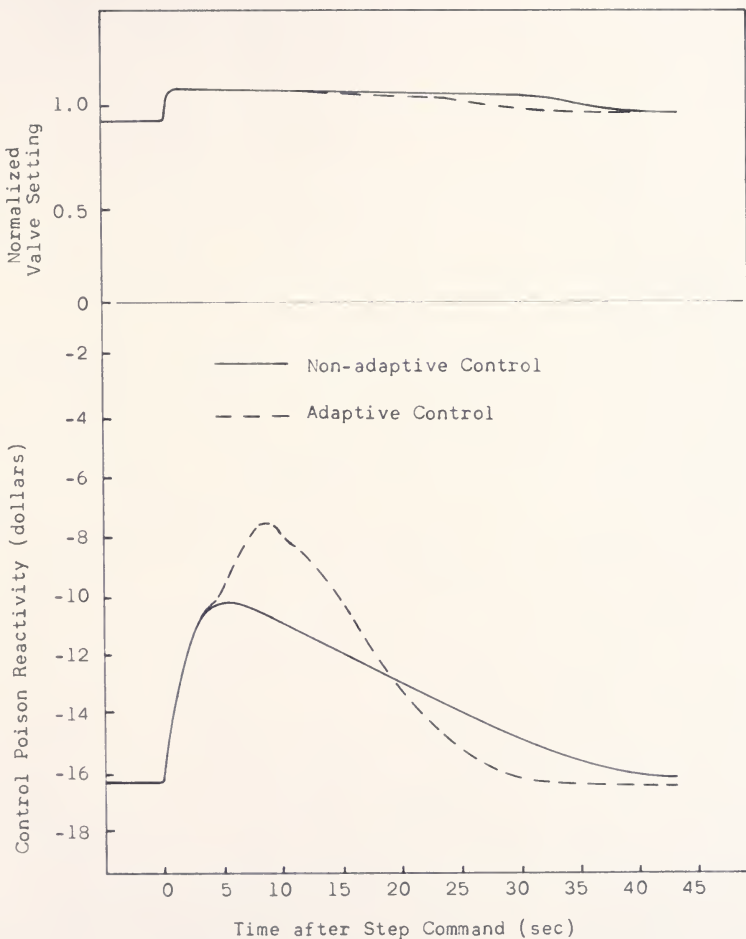


Figure 21. Control Device Response for $2 \tau_p$ (normal)

Figures 12 through 21 represent action by the adaptive control system to upgrade, or speed up, response of the system. Figures 22 and 23 illustrate the performance of the two control systems for the case where the pressure time constant is only 80 per cent of its normal value. This means that the constant-parameter nuclear rocket engine is more responsive to the full power command, and the adaptive controller acts to degrade, or slow down, its response to match that of the reference model. The reactivity contribution of the control poison follows a markedly different pattern in this case.

The results contained in Figures 12 through 23 are summarized in Figure 24. Here, the time required by the nuclear rocket engine to proceed from 20 per cent to 90 per cent of its design temperature is compared for the two modes of control, i.e., constant-parameter versus model reference control. The figure indicates that the reference model requires 14 seconds to make the transition. The constant-parameter system's rise time varies from 14.4 seconds to 29 seconds, or is nearly doubled as the pressure time constant is doubled. The adaptively controlled system's rise time varies from 13.6 seconds to 16.2 seconds for the same variation in

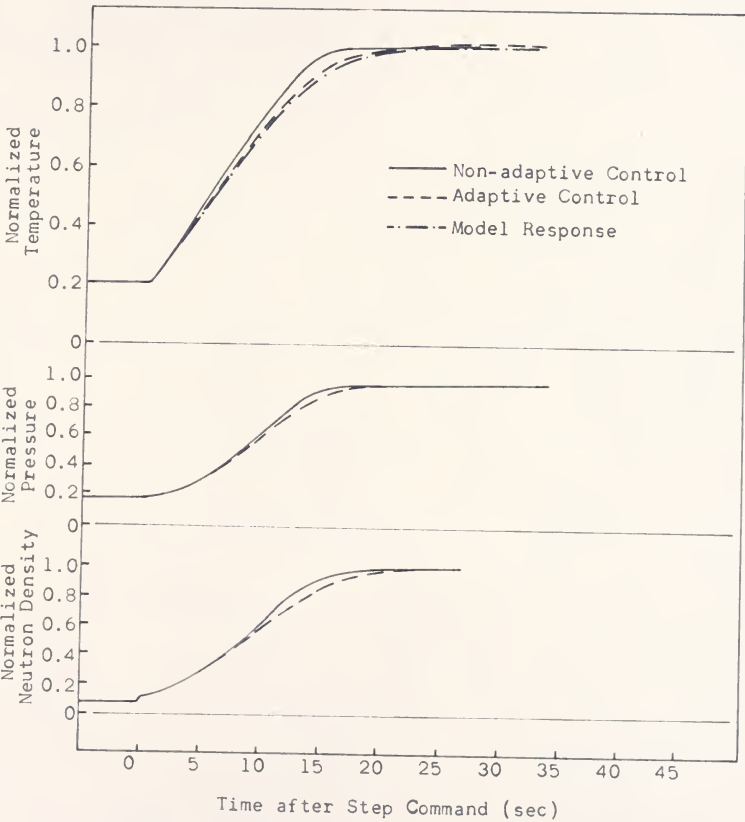


Figure 22. Nuclear Rocket Engine Response for $0.8 \tau_p$ (normal)

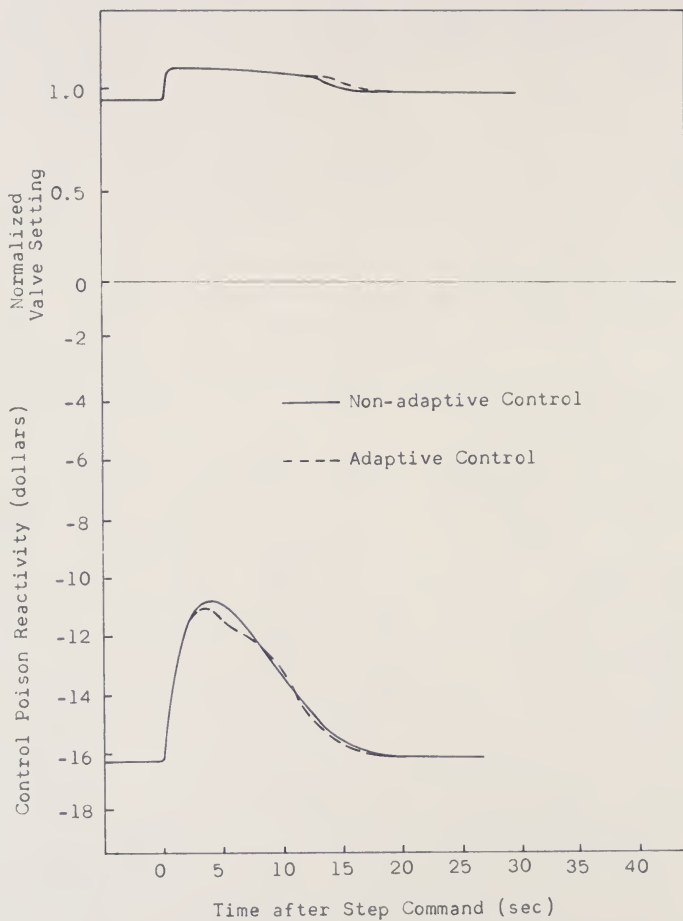


Figure 23. Control Device Response for $0.8 \tau_p$ (normal)

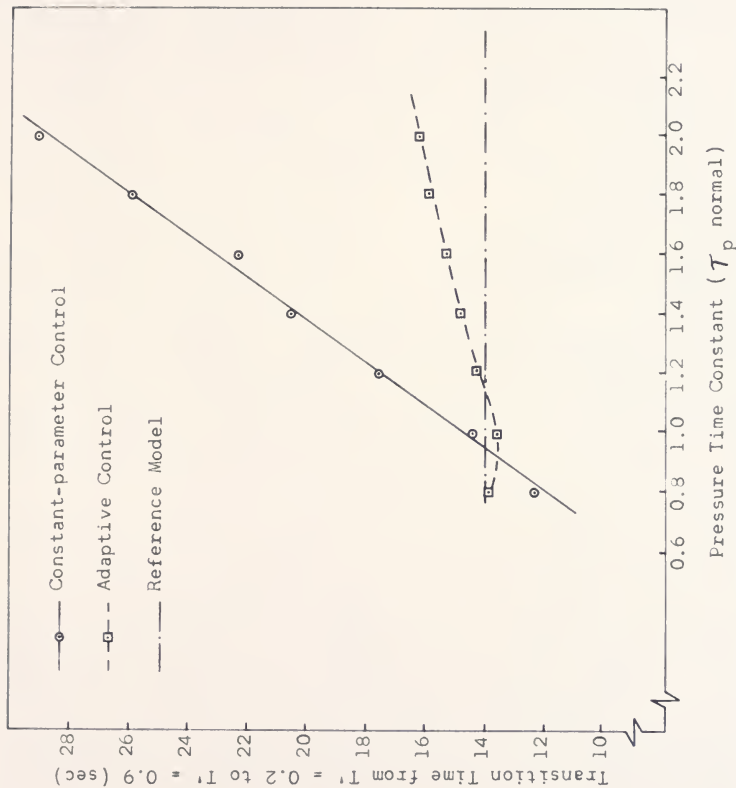


Figure 24. Constant-parameter and Adaptive Control Responses with τ_p Variation

time constant. This amounts to a 19 per cent increase in rise time compared with the normal response. Figure 24 also indicates the degrading action of the adaptive control system in the case of the time constant's being 80 per cent of its expected value.

Figure 24 can also be used as a basis for estimating propellant savings due to action by the adaptive controller. Smith and Stenning determine that the propellant mass rate of flow in the rocket engine is proportional to $P'/(T')^{1/2}$, and that the flow rate at full power is 2300 pounds per second. This means that, at the idling condition, a mass flow rate of approximately 1000 pounds per second is to be expected. If the flow rate increases linearly with power, and this is conservative, one can speak of an average mass flow rate of 1600 pounds per second during the transition from idle to full power. The rocket engine in its "normal" state then uses slightly more than 22,000 pounds of propellant during the transition. In its degraded, or sluggish condition, with the pressure time constant doubled, about 44,000 pounds are expended. Now with the adaptive control system, transition to full power in the sluggish condition is accomplished with about 26,000 pounds of propellant, representing a savings per transition of

18,000 pounds, when compared with performance of the constant-parameter control system. Or, to look at it another way, the savings represents 8 seconds of full power operation.

It is important to observe the variation of the adjustable gain G_2 for these different conditions. Figure 25 shows the adjustment action brought about by the adaptive loop. The gain is expressed as G_2^* , which is equal to $G_2/(2)^{1/2}$, as indicated in equation 4.15. As expected, greatest adjustment is required and executed for the most sluggish condition, i.e., for the doubled pressure time constant.

The variation of G_2^* is particularly interesting in the case of the 80 per cent time constant. Here, it is observed that, for a time, the gain becomes negative. Recall that, in this configuration, the control system is degrading the performance exhibited by the constant-parameter system. This brings up the question of system stability, which is discussed below. It should be noted that the maximum excess reactivity involved in these transitions to full power amounts to 0.6 dollar.

Stability Considerations

The transition for the 80 per cent time constant results in G_2^* being driven negative by the adaptive

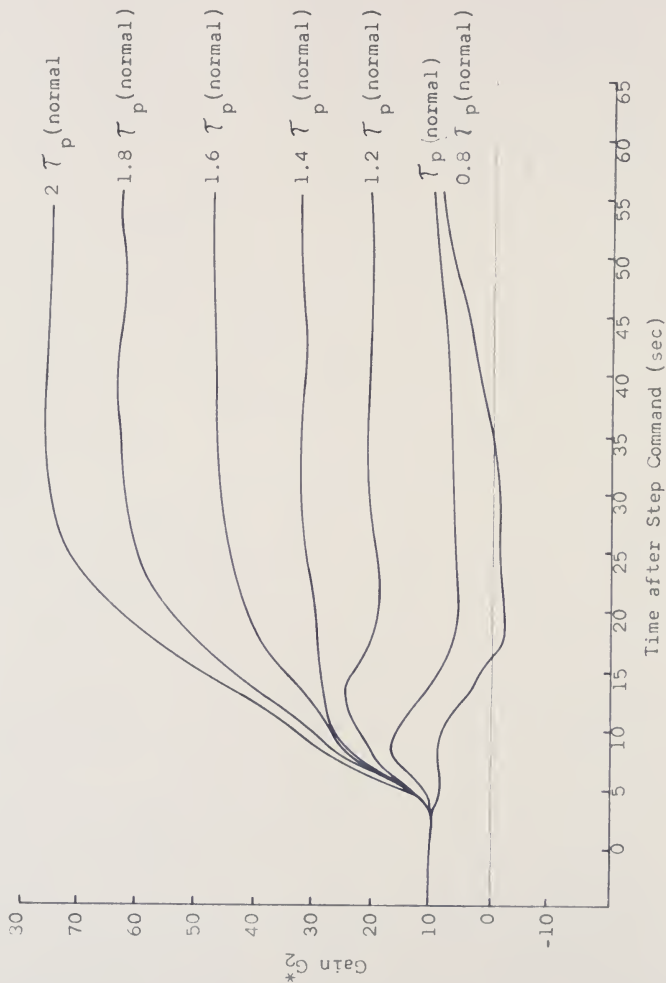


Figure 25. Adaptation of Control Poison Proportional Control Gain

control system. An indication of system instability for negative G_2 , i.e., $(2)^{1/2} G_2^*$, comes from the simplified transfer function of Chapter III. The Routh-Hurwitz stability criterion applied to the transfer function given by equation 3.26 establishes that, for stability of the nuclear rocket engine, $X > 0$, which implies $G_2 > -30.6$, and $Y > 0$, which implies $G_2 > -5.78$. The more restrictive of these two necessary conditions is that $G_2 > -5.78$. It is not advisable to rely too heavily upon a linearized approximation for stability analysis of a nonlinear system, and so the following analog simulation was undertaken for both the constant-parameter system and the adaptively controlled system.

As the nuclear rocket engine makes its transition from idle to full power, a dollar step of positive reactivity is introduced 5 seconds after the transition command. This is accomplished by passing 5 volts to the input of amplifier #20 of Figure 33. Figure 26 depicts the nuclear rocket engine's response.

The constant-parameter control system allows a dangerous temperature overshoot, but succeeds in limiting the temperature surge. Model reference adaptive control appears to handle the added reactivity

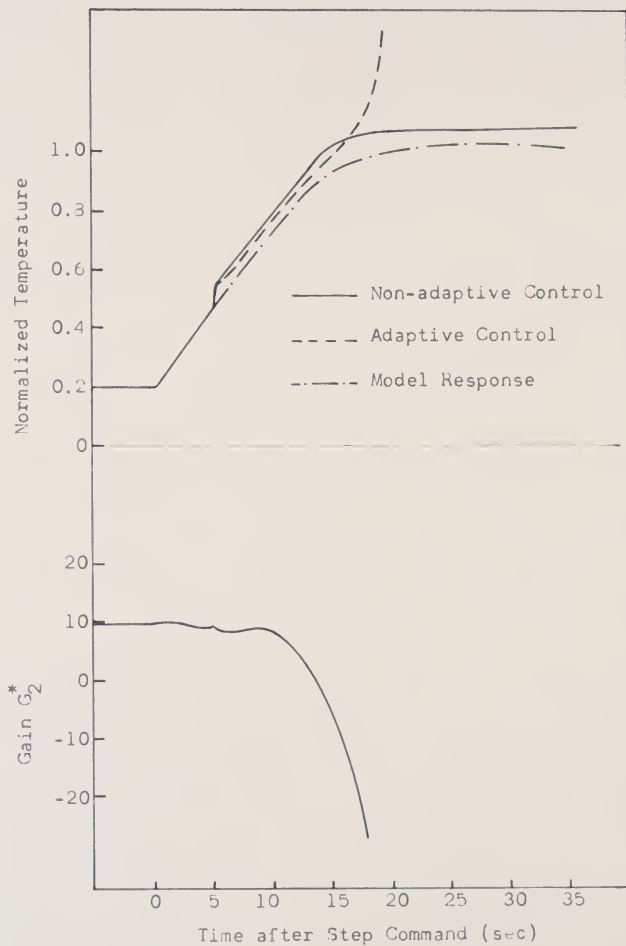


Figure 26. System Instability Due to Negative G_2^*

satisfactorily until the gain G_2^* becomes negative. The system becomes unstable at a G_2^* value of approximately -6.0.

This simulated configuration is one in which the system temperature is greater than called for by the temperature command. The adaptive control system acts to match the system's response with that of the reference model and drives the gain G_2^* negative. This adaptive action results in an unstable situation.

The stability boundary is presented as Figure 27 where values of G_2^* are plotted versus the difference between command and system temperatures. The constant-parameter system was simulated at steady-state design temperature and then disturbed with step signals representing the indicated temperature differences. The gain G_2^* was varied through a small negative range and values leading to unlimited temperature response are used for the stable-unstable boundary. It appears that the constant-parameter system has a limited region of stability for this configuration. A stability map with the system initially at a steady-state temperature of $0.2T_{\text{design}}$ has essentially the same appearance.

Although Figure 27 indicates a stable region for negative G_2^* , this does not mean that acceptable nuclear

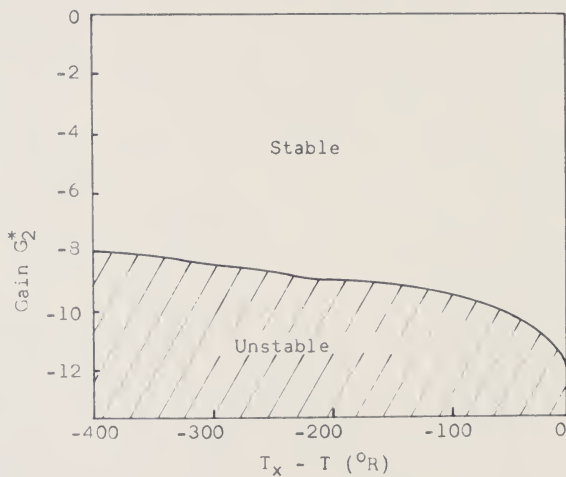


Figure 27. Stability Boundary for Constant-parameter System

rocket engine operation can be attained with these values when the system approaches design temperature. This is due to the fact that intolerable temperature overshoot is encountered for nearly every negative value of G_2^* . This means that, for the nuclear rocket engine under study, negative values for G_2^* are to be avoided, and limiting devices appear to be necessary.

CHAPTER VI

SUMMARY AND CONCLUSIONS

Summary of Results

Model reference parameter adjustment techniques were found effective in controlling a nuclear rocket engine. A relatively simple second-order model (2 poles and 1 finite zero) was suitable as a reference model for the step command inputs utilized in the investigation. Acceptable operation of the nuclear rocket engine was possible with a limited valve opening due to the addition of proportional control of control poison.

The system's pressure time constant was varied to test performance of the adaptive control loop. Acceptable system performance was achieved with adaptive control as the value of this time constant was increased to twice its normal value. Savings in time required for transition from idle to full power and the attendant savings in propellant were found significant when compared with results of constant-parameter control.

Dynamics of the nuclear rocket engine permitted the adaptive system to be more effective in upgrading engine response than in degrading its response. In the latter case, with the gain-control parameter selected for adaptation, it was observed that the adaptive controller could cause unstable operation. This instability could be avoided through use of limiting circuits in the adaptive loop.

Implications of Results

The results of this investigation are encouraging; however, this control concept must be evaluated within the framework of the total body of knowledge pertaining to the development of nuclear rocketry. For example, this study indicates that propellant savings of up to 20,000 pounds per transition from idle to full power are possible with adaptive control. But, is there a mission, and will there be a vehicle, where such savings warrant the addition of a possibly more complicated control system? For the same case, it has been shown that adaptive control reduces the transition time from idle to full power by as much as 14 seconds. Does this shorter rise time substantially reduce the complexity of attitude and guidance control problems so that requirements for elaborate guidance control programming are reduced and the addition of adaptive control is justified?

Finally, would the addition of adaptive control represent a significant increase in system weight and complexity, or can this "cost" be reduced through utilization of computing equipment already on board? These questions cannot be answered here, but as mentioned above, the results of this study are sufficiently encouraging to prompt further investigation of this application of model reference adaptive control.

Of course, it can be reasonably argued that adaptive control should not be considered for application in systems which can be suitably controlled with conventional techniques. Adaptive control does bring with it problems in analysis and requirements for additional equipment. However, such arguments become less significant if the conventional systems require elaborate programming of controller gain changes or depend upon humans for this adjustment when they might possibly be under severe stress, or even incapacitated for periods of time. A nuclearly powered rocket with its crew presents a provocative topic for such discussion.

Suggestions for Further Study

A logical sequel to this work would involve adaptive adjustment of additional parameters. System

response with control emphasis shifted from control poisons to the hydrogen propellant should be studied. Adaptive control should be studied for application in all phases of the rocket engine's operating regime, i.e., startup and shutdown control.

The nuclear rocket engine model should be upgraded within the bounds of available computing equipment. The stability considerations encountered in this study point up the need for some knowledge of the useable range of values for the parameters to be varied in any new engine model. To all of this should be applied recently developed techniques which purport to allow the control system designer to determine compensation for improving adaptation and to predict responses of model reference adaptively controlled systems (24). Finally, consideration should be given to the use of model reference techniques for determination of nuclear reactor system parameters.

APPENDICES

APPENDIX A

PARAMETER ADJUSTMENT ADAPTIVE CONTROL

Osburn's derivation (15) of a model reference adaptive control system design procedure is summarized here for completeness. The problem is "to define optimum performance and provide the means for automatic enforcement of this performance."

System performance is considered optimum when a selected performance index is minimized. The greatly simplified schematic diagram of Figure 28 depicts the quantity which is used to generate the performance index (PI), i.e., the response error $e(s)$ between system and model outputs. The selected PI is given by

$$PI = \int_{t_1}^{t_2} F(e)dt \quad (A.1)$$

where $F(e)$ is an even function of e .

The definition of optimum performance for a system with an adjustable parameter K requires that

$$\frac{\partial (PI)}{\partial K} = \frac{\partial}{\partial K} \int_{t_1}^{t_2} F(e)dt = 0 \quad (A.2)$$

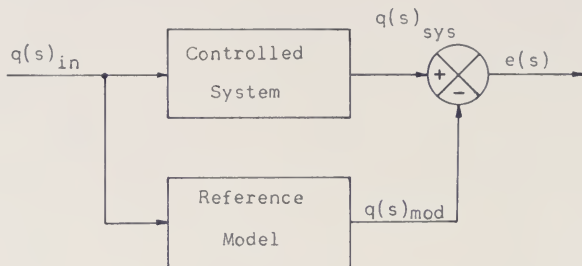


Figure 28. Generation of Response Error in Model Reference Adaptive Control System

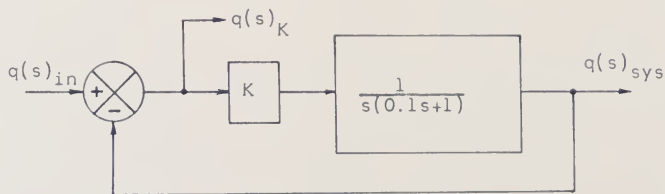


Figure 29. Block Diagram of Second-order System with Adjustable Parameter in Feedforward Path

This leads to the following definition of error quantity (EQ).

$$EQ = \frac{\partial}{\partial K} \int_{t_1}^{t_2} F(e) dt \quad (A.3)$$

Parameter adjustment is then performed according to

$$\Delta K = -S_a(EQ) \quad (A.4)$$

where S_a is the adaptive loop gain. It should be noted that Osburn's derivation provides for adjustment of n parameters; however, this summary describes the derivation for adjustment of only a single parameter.

Equation A.3 is rewritten as

$$EQ = \int_{t_1}^{t_2} \frac{\partial F(e)}{\partial K} dt \quad (A.5)$$

with the assumptions that t_1 and t_2 are independent of the adjustable parameter K and that $\partial F(e)/\partial K$ are continuous functions of both K and t . Now

$$\frac{\partial F(e)}{\partial K} = F'(e) \frac{\partial e}{\partial K} \quad (A.6)$$

$$F'(e) = \frac{dF(e)}{de} \quad (A.7)$$

Methods for obtaining signal indications of these various quantities are now described. As indicated in Figure 28, the response error is defined as

$$e(s) = q(s)_{\text{sys}} - q(s)_{\text{mod}} \quad (A.8)$$

where $q(s)_{\text{sys}}$ is the system output, and $q(s)_{\text{mod}}$ is the model output. Since $q(s)_{\text{mod}}$ is not affected by any parameter of the system,

$$\frac{\partial e(s)}{\partial K} = \frac{\partial q(s)_{\text{sys}}}{\partial K} \quad (\text{A.9})$$

this partial derivative of the system output with respect to the variable parameter K must be obtained.

Osburn shows that generation of $\partial q(s)_{\text{sys}} / \partial K$ requires the use of the system characteristic operator and various derivatives of the system input and output quantities. He points out that

. . . it is exactly the ignorance of detailed information about these coefficients and system characteristics that leads to the necessity for providing the system with a self-adaptive capability.

This apparent impasse is circumvented by noting that the control system can meet its design specifications whenever the variable parameter K is suitably adjusted. Since the model is designed to represent these specifications, Osburn reasons that the model dynamics can provide a good approximation to the system dynamics and model characteristics can be employed to generate an approximation to the needed $\partial q(s)_{\text{sys}} / \partial K$ "in some neighborhood of the system optimum response state." His work has shown that this approach makes possible

a practical and effective design technique. The approximate, or indicated, partial derivative, is called the error weighting function (W_E).

All quantities required for parameter adjustment have been treated, and as is shown below, are available. Equation A.5 is rewritten as

$$EQ = \int_{t_1}^{t_2} F'(e) W_E(t) dt \quad (A.10)$$

and for $F(e) = e^2$, as is the case in Osburn's work and in the present study, equation A.10 becomes

$$EQ = 2 \int_{t_1}^{t_2} e(t) W_E(t) dt \quad (A.11)$$

Equation A.4, together with equation A.11, indicates that the parameter K is adjusted according to

$$\frac{dK}{dt} = -S_a e(t) W_E(t) \quad (A.12)$$

The following example is used to demonstrate Osburn's design technique, and it also points up the availability of signals required for parameter adjustment as called for in equation A.12. Figure 29 is the block diagram of a second-order system with an adjustable parameter in the feedforward path. It is desired to adjust K by using a reference model with the open loop transfer function $10/s(0.1s + 1)$. The design procedure

calls for generation of $\partial q(s)_{\text{sys}} / \partial K$. For the system of Figure 29,

$$q(s)_{\text{sys}} = \frac{K}{s(0.1s + 1) + K} q(s)_{\text{in}} \quad (\text{A.13})$$

The required partial derivative is found to be

$$\frac{\partial q(s)_{\text{sys}}}{\partial K} = \frac{s(0.1s + 1)}{[s(0.1s + 1) + K]^2} q(s)_{\text{in}} \quad (\text{A.14})$$

or equivalently,

$$\frac{\partial q(s)_{\text{sys}}}{\partial K} = (TF_{\text{sys}}) \left(\frac{1}{K} \right) \frac{s(0.1s + 1)}{s(0.1s + 1) + K} q(s)_{\text{in}} \quad (\text{A.15})$$

where TF_{sys} is the closed loop transfer function of the controlled system. In terms of the input signal to K , i.e., $q(s)_K$, equation A.15 can be written as

$$\frac{\partial q(s)_{\text{sys}}}{\partial K} = (TF_{\text{sys}}) \left(\frac{1}{K} \right) q(s)_K \quad (\text{A.16})$$

At this point, Osburn's approximation is introduced, i.e., the typically not-so-well-known transfer function of the controlled system is replaced by the transfer function of the reference model. The error weighting function is now written as

$$W_E = (TF_{\text{mod}}) \left(\frac{1}{K} \right) q(s)_K \quad (\text{A.17})$$

Osburn found it advantageous to replace the $1/K$ term of equation A.17 by unity. This improves the adaptive

response, and division by a varying quantity is not necessary. This modification is employed here and equation A.17 becomes

$$W_E = (TF_{\text{mod}})q(s)K \quad (\text{A.18})$$

Parameter adjustment calls for integration of the product of the response error and the error weighting function, according to equation A.12. Osburn's design procedure was derived with the assumption that parameter variation during adaptive response by the system is zero. The nonlinear nature of the system requires this assumption, which is obviously not true in an actual control system. Willis (25) investigated this matter and found that the parameter variation "seen" by the adaptive system is an "apparent" parameter variation which is "very small compared to the actual parameter variation," thus establishing the validity of the assumption.

Figure 30 is a block diagram depicting all components of the example system with its adaptive control loop. The system was simulated on an analog computer and Figure 31 illustrates the adaptive response of the system for two offset values of the variable gain K . No further analysis is presented, since this material is provided to describe the basis for the adaptive loop design contained in Chapter IV.

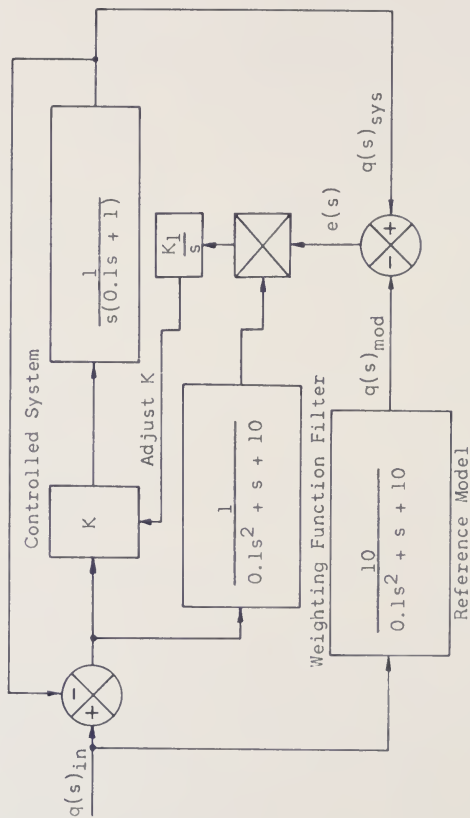


Figure 30. Block Diagram of Second-order System with Adaptive Control Loop

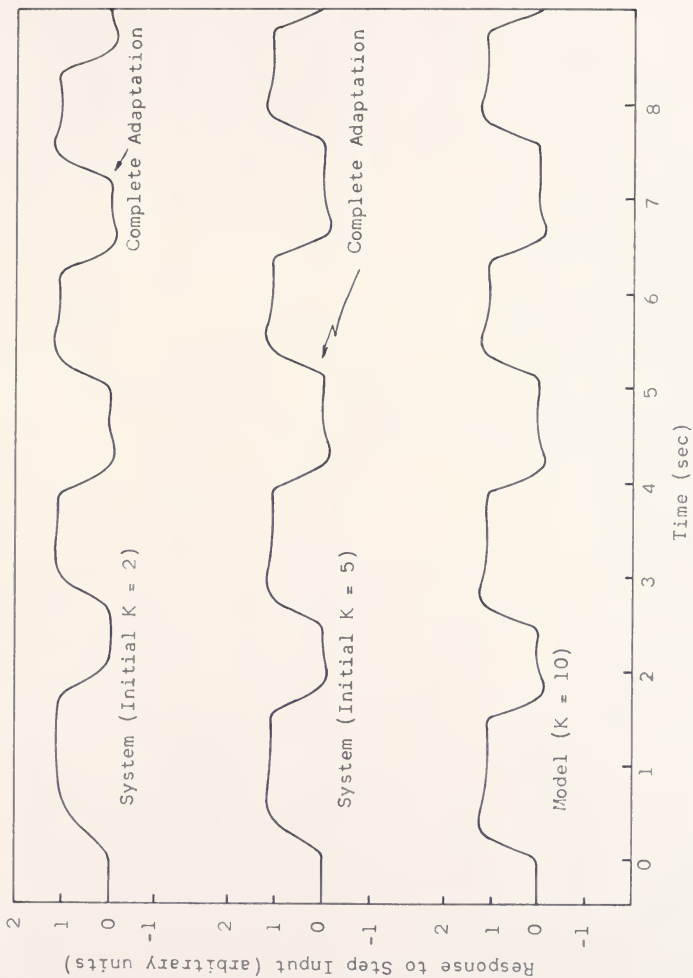


Figure 31. Adaptive Response of Second-order System

APPENDIX B

EQUATION NORMALIZATION

Reactor Kinetics Equations

The kinetics equations used by Smith and Stenning to describe the time behavior of the neutron and precursor populations for a point reactor model are written in the following form:

$$\frac{dN}{dt} = \frac{\rho - \beta}{\Lambda} N + \sum_i \lambda_i C_i \quad (\text{B.1})$$

$$\frac{dC_i}{dt} = \frac{\beta_i}{\Lambda} N - \lambda_i C_i \quad (\text{B.2})$$

Since the magnitudes of quantities in the above equations differ considerably and vary over a wide range, some type of normalization is advantageous for analog computer simulation. In the problem under discussion, all quantities are normalized to design point conditions, i.e., maximum core surface temperature of 5000 °R and inlet stagnation pressure of 1500 psi. This specification of design point conditions solely in terms of temperature and pressure points up the fact that all other variables are dependent and are determined by the

temperature and pressure command inputs to the system. Normalization of equations B.1 and B.2 is accomplished through introduction of the following variables:

$$N' = \frac{N}{N_d} \quad (\text{B.3})$$

$$C' = \frac{C}{C_d} \quad (\text{B.4})$$

In the case of reactivity, normalization amounts to expression of reactivity in dollars, i.e.,

$$\rho' = \frac{\rho}{\beta} \quad (\text{B.5})$$

For the one delayed group approximation employed in this study and for the reactor system in equilibrium at the design point, equation B.2 gives

$$\lambda = \frac{\beta}{1} \frac{N_d}{C_d} \quad (\text{B.6})$$

Substitution of equations B.3, B.4, B.5, and B.6 into equation B.1 leads to equation 2.1.

Similarly, equation B.6 gives

$$\frac{\beta}{1} = \lambda \frac{C_d}{N_d} \quad (\text{B.7})$$

Substitution of equations B.3, B.4, and B.7 into B.2 leads to equation 2.2.

Temperature and Pressure Equations

The temperature equation derived by Smith and Stenning comes from an energy balance where the time rate of change of core thermal energy is given as the difference in the rate at which energy is released in fission and the rate at which energy is removed by the coolant. By developing certain proportionalities, they arrived at the following expression:

$$Mc \frac{T_r}{T} \frac{dT}{dt} = K_1 N - K_2 P(T)^{1/2} \quad (B.8)$$

where the ratio T_r/T is constant. Consideration of the reactor system in equilibrium at the design point leads to

$$K_1 = \frac{K_2 P_d (T_d)^{1/2}}{N_d} \quad (B.9)$$

Additional variables are defined as follows:

$$T' = \frac{T}{T_d} \quad (B.10)$$

$$P' = \frac{P}{P_d} \quad (B.11)$$

Substitution of equations B.3, B.9, B.10, and B.11 into equation B.8 results in equation 2.3, where

$$\tau_t = \frac{Mc (T_d)^{1/2}}{K_2 P_d} \left(\frac{T_r}{T} \right) \quad (B.12)$$

The value for the temperature time constant expressed in equation B.12, based upon the Smith-Stenning work, is included in Table 1. A similar development, based upon conservation of energy, was pursued by Smith and Stenning to obtain the pressure relationship given by equation 2.4.

Reactivity and Controller Equations

Reactivity of the nuclear rocket engine is dependent upon core temperature, propellant density in the core, and control poison, as expressed in the following equation:

$$\rho = \rho_c + \alpha_t T + \alpha_h \frac{P}{T} \quad (\text{B.13})$$

Substitution of equations B.5, B.10, and B.11 into equation B.13 leads to equation 2.10, where

$$\alpha'_t = \frac{\alpha_t T_d}{\beta} \quad (\text{B.14})$$

and

$$\alpha'_h = \frac{\alpha_h P_d}{\beta T_d} \quad (\text{B.15})$$

Normalization of the controller equations was performed in a similar manner.

APPENDIX C

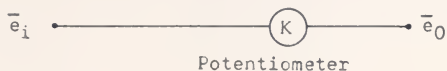
ANALOG SIMULATION

Equipment

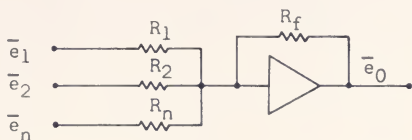
An Applied Dynamics AD-80 analog computer was used for system simulation in this investigation. This machine contains 80 operational amplifiers, 80 coefficient potentiometers, 16 electronic quarter-square multipliers, 8 diode function generators, and 20 special diode networks. With its control logic system, it is capable of temporarily storing computed variables and/or automatic switching and sequencing of computing components, and can perform all functions required for hybrid computation.

Analog Circuit Symbolism

Diagrams of the various analog computer circuits used in this research employ the symbols depicted in Figure 32. The figure also presents the operations which are mechanized by the computer components. Several points should be noted concerning the computer diagrams which follow. The values of potentiometer settings (K) are contained in tables rather than in the computer diagram.

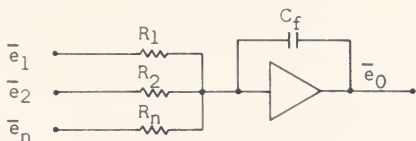


$$\bar{e}_0 = K \bar{e}_i$$



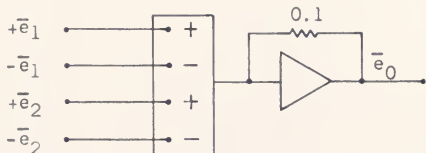
Summing Amplifier

$$\bar{e}_0 = - \sum_{j=1}^n \frac{R_f}{R_j} \bar{e}_i$$



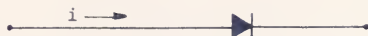
Integrating Amplifier

$$\bar{e}_0 = - \int_0^t \sum_{j=1}^n \frac{C_f}{R_j} \bar{e}_i dt$$



Quarter-square Multiplier

$$\bar{e}_0 = - \frac{\bar{e}_1 \bar{e}_2}{100}$$



Diode

Figure 32. Symbols Used in Analog Computer Diagrams

Only an identification number is used in the computer diagram. Diodes are used in limiting circuits and in the square root circuit required for simulation of the nuclear rocket engine.

Input and feedback resistances for summing amplifiers, and input resistances for integrating amplifiers are expressed in megohms. Values of either 0.1 or 1.0 megohm are used. Feedback capacitances of integrating amplifiers are expressed in microfarads, with values of 0.1 or 1.0 microfarad being used in this research. An identification number is assigned each amplifier in the computer circuit diagrams.

Changing the feedback resistor of a quarter-square multiplier from 0.1 to 1.0 megohm increases the output by a factor of 10. Reversing the inputs of either voltage, i.e., $\pm \bar{e}_1$, or $\pm \bar{e}_2$, reverses the polarity of the multiplier output. These techniques are used in simulation of the nuclear rocket engine as noted in the computer diagrams.

Certain combinations of quarter-square multipliers and summing amplifiers are used to form division and square root circuits. When a voltage $\bar{e}_1$ is divided by $\bar{e}_2$, the output of the division circuit is $100\bar{e}_1/\bar{e}_2$. If the square root of $\bar{e}_1$ is taken, output of the square root circuit is $10(\bar{e}_1)^{1/2}$. Details of these specialized circuits are in-

cluded in the computer circuit diagrams which follow. With a symbolism defined, it is now possible to describe the various simulations performed in the course of this research.

Problem Scaling

Amplitude and/or time scaling is usually a necessary part of analog computer work. Ashley (26) suggests the following scale factor equation for relating machine voltages to problem variables:

$$x = \frac{|x|_{\max}}{\bar{C}_{\max}} \bar{x} \quad (C.1)$$

where x is the problem variable, $|x|_{\max}$ is the largest expected absolute value of the problem variable, $\bar{x}$ is the machine voltage representing x , and $\bar{C}_{\max}$ is the maximum operating voltage for the computing equipment. As noted previously, $\bar{C}_{\max}$ for this study is ± 100 volts. Throughout this study, machine variables are expressed in volts. A bar drawn above a symbol is used to denote these quantities.

The following equations define the scale factors used for simulation of the nuclear rocket engine. Since the primed quantities are dimensionless, scale factors are expressed in reciprocal volts.

$$N^i = \frac{20}{100} \bar{N} \quad (C.2)$$

$$C^i = \frac{20}{100} \bar{C} \quad (C.3)$$

$$T^i = \frac{2}{100} \bar{T} \quad (C.4)$$

$$(T^i)^{1/2} = \frac{(2\bar{T})^{1/2}}{10} \quad (C.5)$$

$$P^i = \frac{4}{100} \bar{P} \quad (C.6)$$

$$\rho^i = \frac{2}{100} \bar{\rho} \quad (C.7)$$

$$\rho_c^i = \frac{20}{100} \bar{\rho}_c \quad (C.8)$$

$$\rho_{cx}^i = \frac{20}{100} \bar{\rho}_{cx} \quad (C.9)$$

$$V^i = \frac{3}{100} \bar{V} \quad (C.10)$$

$$V_x^i = \frac{3}{100} \bar{V}_x \quad (C.11)$$

No time scaling is performed, i.e., machine time equals real time.

With the above amplitude scaling, equations 4.1 - 9 become:

$$\dot{\bar{N}} = 100 \left(\frac{\bar{\rho} \bar{N}}{100} \right) - 50 \bar{N} + 50 \bar{C} \quad (C.12)$$

$$\dot{\bar{C}} = 0.1 (\bar{N} - \bar{C}) \quad (C.13)$$

$$\dot{\bar{T}} = 14.71 \bar{N} - 4.16 \frac{\bar{P} \bar{T}}{10}^{1/2} \quad (C.14)$$

$$\dot{\bar{P}} = 1.7 \frac{\bar{V}(\bar{P} \bar{T}^{1/2}/10)}{100} - 11.32 \frac{(10\bar{P}/\bar{T})(\bar{P} \bar{T}^{1/2}/10)}{100} \quad (C.15)$$

$$\bar{\rho} = 10 \bar{\rho}_c - \bar{T} + 180(10\bar{P}/\bar{T}) \quad (C.16)$$

$$\begin{aligned} \bar{\rho}_{cx} = G_{1/2}^{1/2} \int_0^t (\bar{T}_x^{1/2} - \bar{T}^{1/2}) dt + G_{2/2}^{1/2} \\ (\bar{T}_x^{1/2} - \bar{T}^{1/2}) + \bar{\rho}_{cx}(0) \end{aligned} \quad (C.17)$$

$$\dot{\bar{\rho}}_c = 0.5(\bar{\rho}_x - \bar{\rho}_c) \quad (C.18)$$

$$\begin{aligned} \bar{V}_x = G_3(1.33) \int_0^t (\bar{P}_x - \bar{P}) dt + G_4(1.33) \\ (\bar{P}_x - \bar{P}) + \bar{V}_x(0) \end{aligned} \quad (C.19)$$

$$\dot{\bar{V}} = 5(\bar{V}_x - \bar{V}) \quad (C.20)$$

The grouping of machine variables in the above equations is a consequence of the nature of the manner in which multiplication and division are performed by the analog computer. Equation C.12 contains the quantity

$\bar{P}\bar{V}/100$. This is precisely the output (in volts) obtained when a voltage $\bar{P}$ is multiplied by a voltage $\bar{V}$. Equation 4.19 presents a substitution for this quantity and is accompanied by other definitions which permit the system equations to be handled in a more orderly fashion.

Initial conditions must be determined for the nuclear rocket engine simulation. The "idle" condition is considered to be that condition corresponding to a temperature of 1000°R and pressure of 275 psi. This is taken as the initial condition, and the scale factor equations together with the system equations at steady state, i.e., all derivatives zero, yield the initial condition values of Table 2.

With the system equations scaled, and initial conditions determined, it is possible to proceed with the analog computer simulation of the nuclear rocket engine.

Constant-Parameter System Simulation

Analog computer circuits for simulating the constant-parameter nuclear rocket engine are represented in Figures 33 and 34. They depict a straightforward mechanization of equations 4.10 - 23, however the following points require discussion.

TABLE 2
INITIAL CONDITIONS

Temperature		
T'	.	0.2
$\bar{T}$		10.0 volts
Pressure		
P'		0.183
$\bar{P}$		4.583 volts
Neutron Density		
N'		0.082
$\bar{N}$		0.410 volts
Precursor Density		
C'		0.082
$\bar{C}$		0.410 volts
Reactivity		
ρ'		0.0 dollars
$\bar{\rho}$		0.0 volts
Mechanical Poison Reactivity		
ρ'_c		-16.28 dollars
$\bar{\rho}_c$		-81.4 volts
Valve Setting		
V'		0.915
$\bar{V}$		30.5 volts

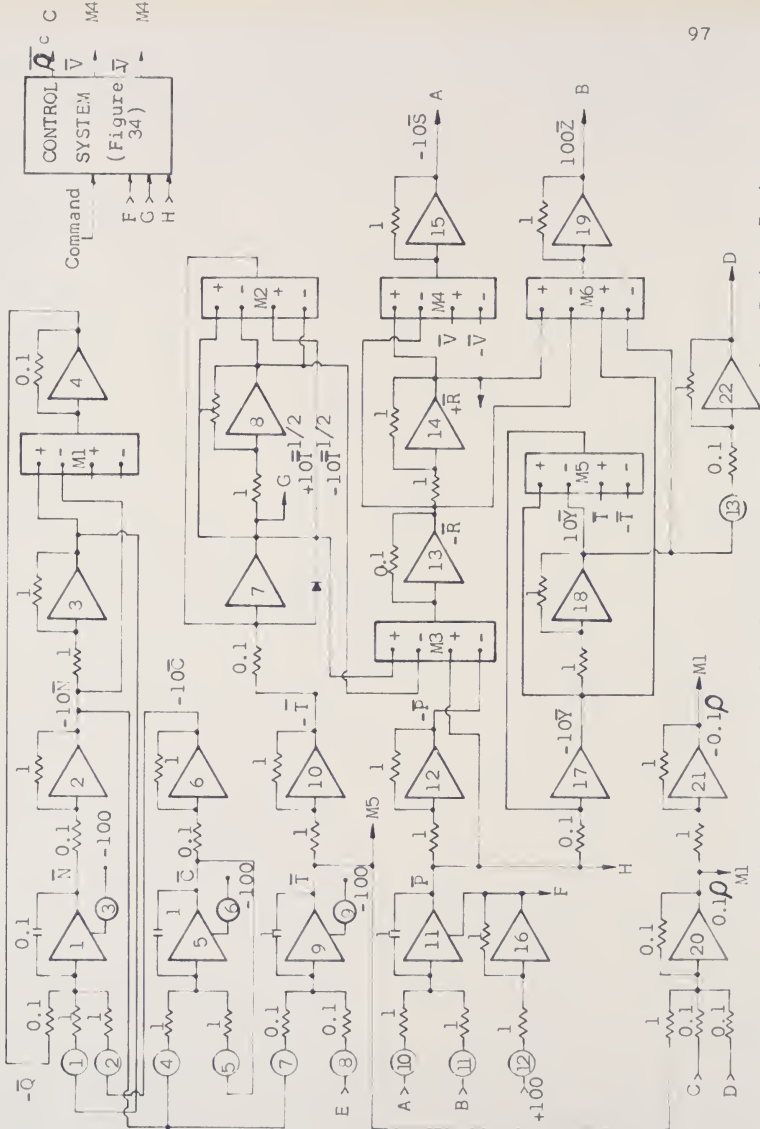


Figure 33. Analog Computer Diagram for Constant-parameter Nuclear Rocket Engine

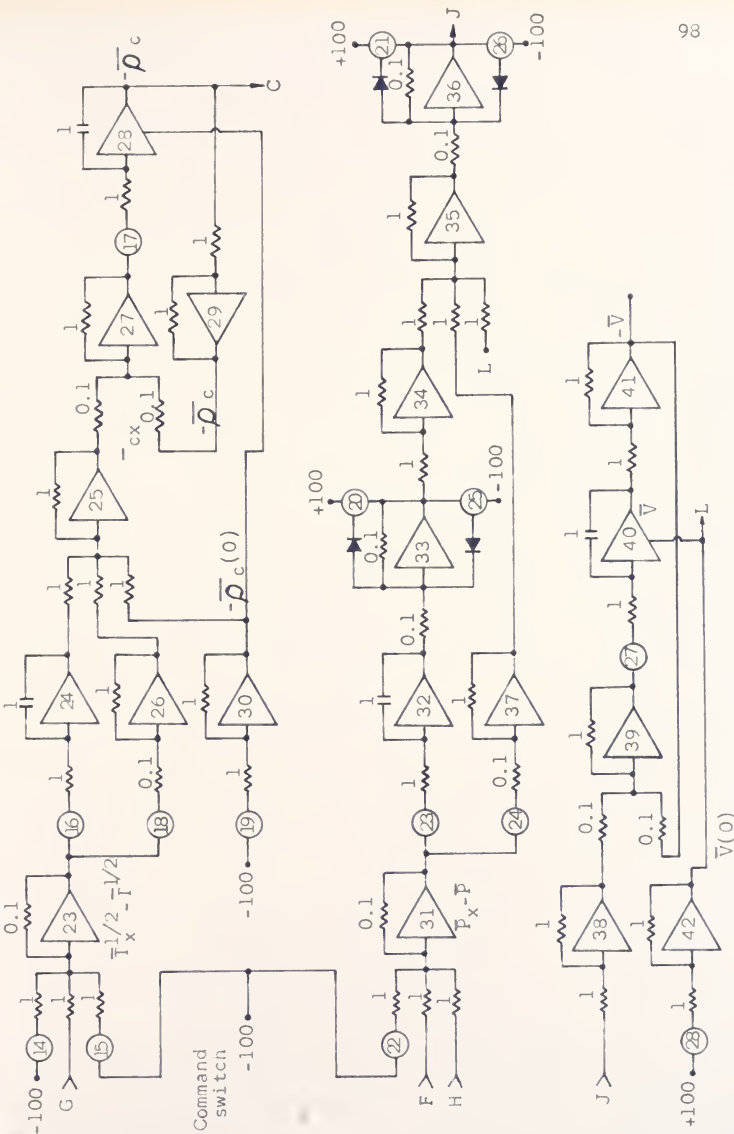


Figure 34. Analog Computer Diagram for Constant-parameter Control System

Amplifier #20 of Figure 33 indicates generation of $0.1\bar{p}$ as opposed to $1.0\bar{p}$. The $180\bar{V}$ contribution to $\bar{p}$ is the output of the division circuit involving amplifiers #17 and #18. This division circuit proved to be noisy, and better performance of the simulator was attained with its output reduced by a factor of ten. The resulting reactivity voltage is passed to multiplier #M1 where it is multiplied by $10\bar{N}$, thereby regaining the factor of ten. This minimizes noise problems in the $\bar{Q}$ output.

As mentioned above, this study is based upon the concept of limiting the control valve setting to a value 10 per cent greater than the design valve setting at full power. This is achieved by placing this constraint on the valve command $\bar{V}_x$. The limiting circuit involving amplifier #16 accomplishes this, while another limiting circuit involving amplifier #33 is necessary to prevent saturation of amplifier #25.

Response of the simulated constant-parameter nuclear rocket engine is contained in Figure 7. Potentiometer settings for this transition from idle to full power are listed in Table 3. Computer setup and problem scaling were checked with MIMIC (27), a digital simulator program, which is described in Appendix D.

TABLE 3

POTENTIOMETER SETTINGS FOR CONSTANT-PARAMETER ENGINE
SIMULATION

Potentiometer	Setting
1	0.50
2	0.50
3	0.0041
4	0.01
5	0.10
6	0.0041
7	0.1471
8	0.416
9	0.10
10	0.17
11	0.1132
12	0.0458
13	0.18
14	0.316
15	0.391
16	0.354
17	0.50
18	1.00

TABLE 3 (Continued)

Potentiometer	Setting
19	0.8109
20	0.858
21	0.25
22	0.2042
23	0.0266
24	0.94
25	0.80
26	0.50
27	0.50
28	0.306

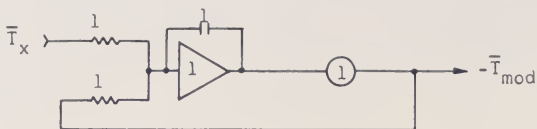
Reference Model Simulation

Three reference models were considered for use in adaptively controlling the nuclear rocket engine. These included a first-order model, a "pure" second-order model, and a second-order model with a zero. Transfer functions for the three models are given by equations 4.24, 4.25, and 4.26. Part a, part b, and part c of Figure 35 are analog computer diagrams for the models. The diagrams point up increasing requirements for computing components as the models become more complex. Potentiometer settings used in the simulations are listed in Table 4.

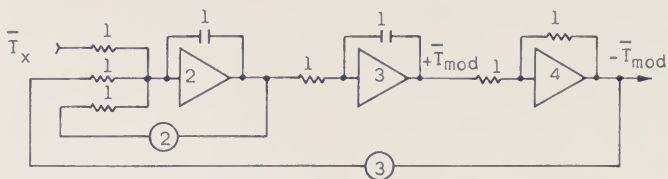
A discussion of the transient response for each model is included in Chapter IV. The second-order model with a zero was chosen for adaptive control of the nuclear rocket engine.

Adaptive Loop Simulation

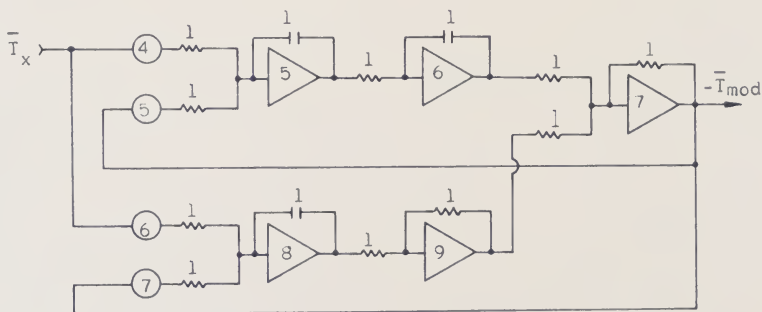
Figure 36 is the analog computer diagram for the adaptive loop, and Table 5 contains the corresponding potentiometer settings. The block diagram for the loop is presented in Chapter IV. Figures 33, 34, and 36 constitute the complete analog computer diagram for the adaptively controlled nuclear rocket engine. Integrating



(a) First-order Model



(b) "Pure" Second-order Model



(c) Second-order Model with a Finite Zero

Figure 35. Analog Computer Diagrams for Reference Models

TABLE 4
POTENTIOMETER SETTINGS FOR MODEL SIMULATIONS

Model	Potentiometer	Setting
First-order	1	0.10
"Pure" Second-order	2	0.251
	3	0.032
Second-order with Zero	4	0.0256
	5	0.0256
	6	0.0494
	7	0.2369

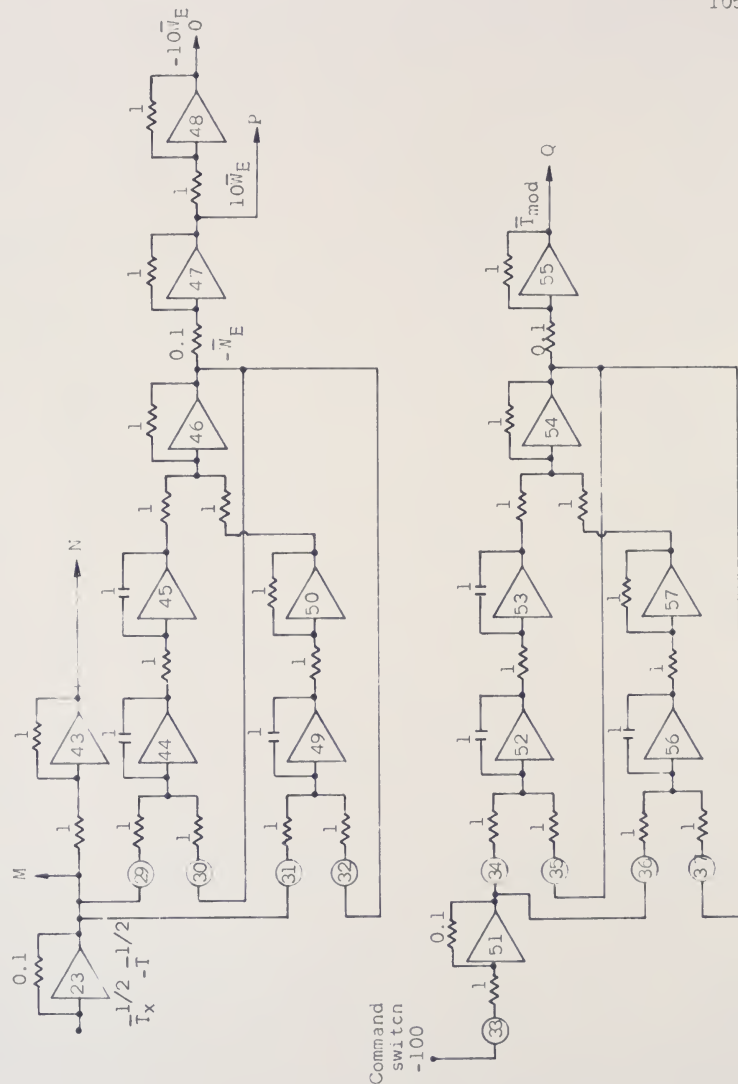


Figure 36. Analog Computer Diagram for Adaptive Loop

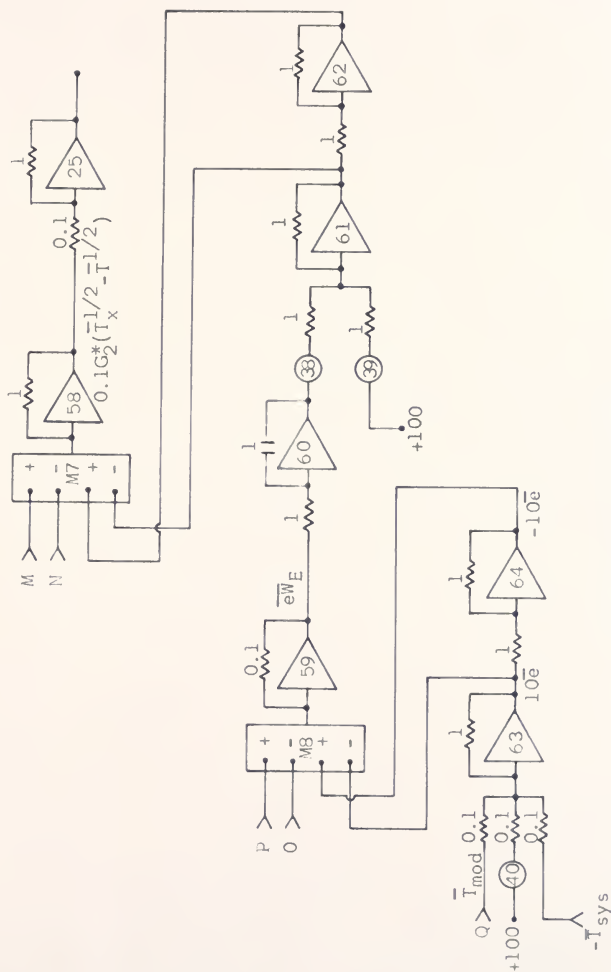


Figure 36 (Continued)

TABLE 5
POTENTIOMETER SETTINGS FOR ADAPTIVE LOOP
SIMULATION

Potentiometer	Setting
29	0.0256
30	0.0256
31	0.0494
32	0.2369
33	0.40
34	0.0256
35	0.0256
36	0.0494
37	0.2369

amplifiers, summing amplifiers, potentiometers, and quarter-square multipliers are numbered consecutively through the three figures. Note that the adaptive loop of Figure 36 replaces potentiometer #18 and summing amplifier #26 of Figure 34. This means that 63 amplifiers, 40 potentiometers, and 8 multipliers were required for simulation of the nuclear rocket engine. In addition, one potentiometer, and one summing amplifier were used to generate the time base for the X-Y recorder, hence a total of 64 amplifiers and 41 potentiometers were required for the problem. The following discussion is presented to aid in interpretation of the adaptive loop analog computer diagram.

The input signal to the variable gain, i.e., the output of amplifier #23 is passed through the weighting function filter which has the same transfer function as the reference model. This filtered signal $\bar{W}_E$ is increased by a factor of ten in amplifier #47, and is passed to multiplier #M8. Similarly, the temperature command signal is passed to the reference model. Potentiometer #34 allows a temperature command of 40 volts which is reduced by a factor of ten in amplifier #51. This reduction is necessary to avoid saturation of reference model integrators. Forty

volts corresponds to the command calling for transition from idle to full power. The output of amplifier #54 is $0.1 \bar{T}_{\text{mod}}$, and multiplication by 10 in amplifier #55 gives $\bar{T}_{\text{mod}}$. An error signal is formed by subtracting model temperature from system temperature in amplifier #63. The 10-volt input to amplifier #63 from potentiometer #41 corresponds to the command to idle at a temperature of 1000°R . Generation of ten times the error as opposed to the error itself is discussed below.

Inputs $\pm 10e$ and $\pm 10\bar{W}_E$ to multiplier #M8 with polarity reversed in one case results in $\bar{e}\bar{W}_E$. Integrating this signal produces the desired adjustment of the variable gain G_2 . Potentiometer #39 represents the initial gain G_2 . Passing these signals to amplifier #25 through multiplier #M7 completes the adaptive loop.

Generation of $10e$ as mentioned above is dictated by the nature of electronic quarter-square multipliers. These devices employ diode squaring networks which essentially approximate the square of a function by a series of straight lines. Operation of the multipliers with inputs less than 5 volts does not usually produce a reliable output. This means that adaptive action, which depends upon the produce $\bar{e}\bar{W}_E$, lags a bit, waiting for a sizable enough error signal to cause reliable multiplier

operation. Increasing the error signal by a factor of 10 noticeably speeds up the initiation of adaptive action.

One final point should be mentioned with regard to simulation of the adaptive loop. Note that the command given to the rocket system, or "plant," is of the form $\bar{T}_x^{1/2}$, and it is this type of signal which is passed to the weighting function filter. On the other hand, a direct temperature command $\bar{T}_x$ is passed to the reference model. This does not violate Osburn's design procedure, for as he has observed,

. . . the sign of the weighting function is more important than its magnitude, since so long as the sign is correct the proper polarity of corrective action is indicated.

With this in mind, the present study utilized the control system of Smith and Stenning, but employed a simpler command signal for the reference model. Simulation indicated that this arrangement results in acceptable adaptive action.

APPENDIX D

DIGITAL CHECK

It is desirable in any analog computer simulation to have a means for checking results. In this study, digital computing equipment at the University of Florida Computing Center was used to solve the original rocket engine equations, i.e., equations 2.1 through 2.9 for certain values of the system's parameters. This solution was compared with the results of a corresponding analog simulation. Solutions from the two computers were in good agreement, thereby confirming the accuracy of the analog computer setup, and problem scaling.

A digital computer program called MIMIC (27) was used to solve the rocket engine equations. MIMIC, a digital simulator program was developed for use on an IBM 709C/7094 computer with a Fortran IV IBSYS monitor. University of Florida Computing Center personnel modified the program for use with the IBM 709 computer.

The following example illustrates that MIMIC's input language endows the digital computer with the parallel nature of the analog computer. Suppose that the following equation is to be solved.

$$\ddot{Y} + \dot{Y} + Y = 0 \quad (D.1)$$

where $\dot{Y}(0) = 4$, and $Y(0) = 0$. With an analog computer, the problem solver typically solves for the highest ordered derivative as follows:

$$\ddot{Y} = -\dot{Y} - Y \quad (D.2)$$

Then, assuming that the second order derivative is available, integrating and summing amplifiers are connected so that the solution Y is obtained.

Figure 37 illustrates the format in which MIMIC instructions would be punched on an IBM card for the same problem. "Results" are punched beginning in column 10, with "expressions" beginning in column 19. The variable names 2DX and 1DX represent the second and first derivatives respectively, while Y represents the desired solution. The INT function integrates the first argument with respect to the independent variable, and the initial condition is the second argument. MIMIC employs a high order Runge-Kutta subroutine for integration. Note that the MIMIC instructions proceed very much like the corresponding analog computer program. With appropriate control instructions, the solution Y may be obtained as a function of the independent variable.

10 RESULT	19 EXPRESSION
2DY	-1DY-Y
1DY	INT(2DY, 4.)
Y	INT(1DY, 0.)

Figure 37. MIMIC Instructions

For the digital check mentioned above, which did not include proportional control of control poison, the MIMIC Source-language Program was as follows:

```

N      INT(50.*(RHO*N-N+C),.08199)
C      INT(0.1*(N-C),.08199)
TP     INT(1.471*(N-P*STP),.2)
STP    SQR(TP)
P      INT(0.4*(P*V*STP-P*P/STP),.18333)
RHO    RHOC-TP+18.*E/TP
RHOX   INT(0.5*(1.-STP),-16.3)
RHOC   INT(0.5*(RHOX-RHOC),-16.3)
VX     INT(0.002*(1.-P),.91665)+0.7*(1.-P)
V      INT(5.*(VX-V),.91665)
HDR(T,TP,P,N,C,RHO)
HDR(  ,  ,RHOC,RHOX,V,VX)
HDR
OUT(T,TP,P,N,C,RHO)
OUT(  ,  ,RHOC,RHOX,V,VX)
OUT
FIN(T,50.)
END

```

MIMIC contains a large number of function sub-routines, including logical functions. The ease with which its instructions can be presented and its great similarity to analog programming make it an ideal instrument for checking analog computer scaling and setup.

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BIOGRAPHICAL SKETCH

Jack Thomas Humphries was born May 7, 1929, at Middlesboro, Kentucky. He graduated from Corbin High School in June, 1947, and received the Bachelor of Science degree in physics from the University of Kentucky in June, 1951. Immediately following his graduation, he worked as a physicist in the Materials Laboratory at Wright-Patterson Air Force Base.

He was called to active duty in the United States Air Force in January, 1953, and served as a nuclear research officer at Wright Air Development Center until August, 1956. In March, 1958, he received the Master of Science degree in nuclear engineering from the Air Force Institute of Technology and was assigned to the faculty of the Air Force Academy, Colorado. He served, in turn, as an instructor of physics and an assistant professor of mathematics. He began graduate study under Air Force auspices at the University of Florida in June, 1963, and until the present time has pursued work toward the degree of Doctor of Philosophy.

Jack Thomas Humphries is the son of Anna M. Humphries and the late Thomas C. Humphries. His wife is the former Mildred Alice Eaton and he is the father of two children. He is a career officer in the United States Air Force and is a member of the American Nuclear Society, the American Society for Engineering Education, Tau Beta Pi, and Phi Kappa Phi.

This dissertation was prepared under the direction of the chairman of the candidate's supervisory committee and has been approved by all members of that committee. It was submitted to the Dean of the College of Engineering and to the Graduate Council, and was approved as partial fulfillment of the requirements for the degree of Doctor of Philosophy.

April, 1966

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